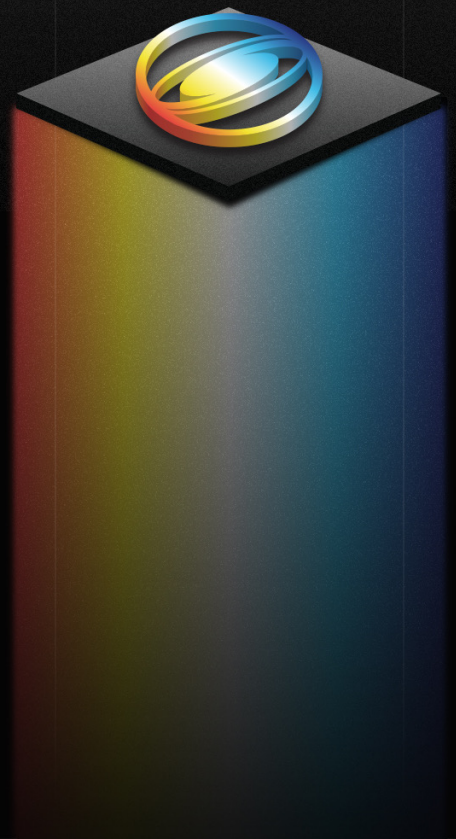
SIGGRAPH 2024
DENVER+ 28 JUL — 1 AUG

THE PREMIER CONFERENCE & EXHIBITION ON COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

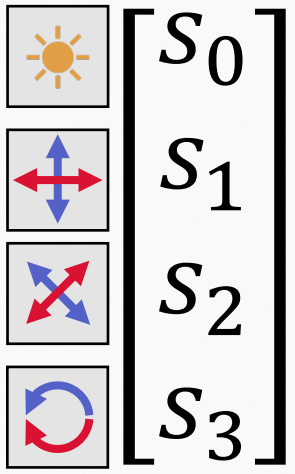
Spin-Weighted Spherical Harmonics for Polarized Light Transport



SHINYOUNG YI DONGGUN KIM JIWOONG NA
XIN TONG MIN H. KIM

KAIST
Microsoft Research 微软亚洲研究院

Polarization in Stokes parameters



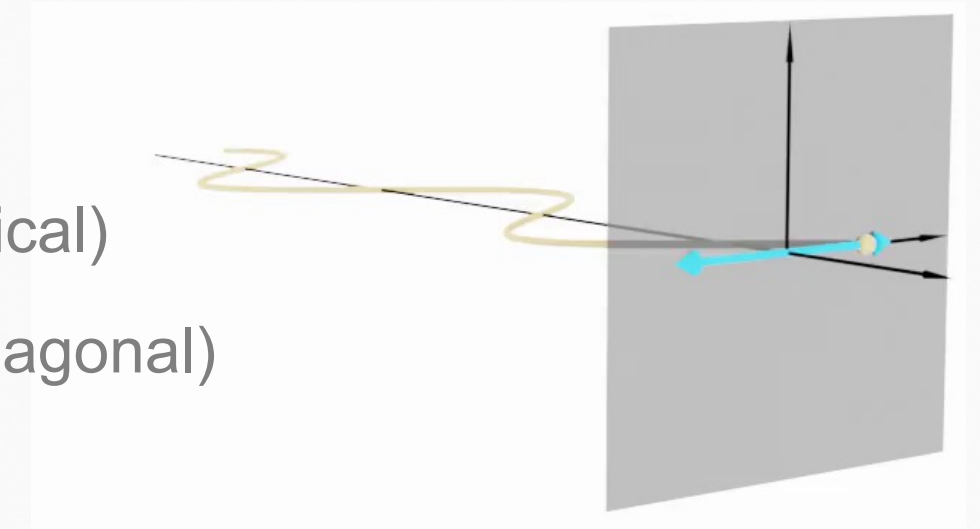
$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

total intensity

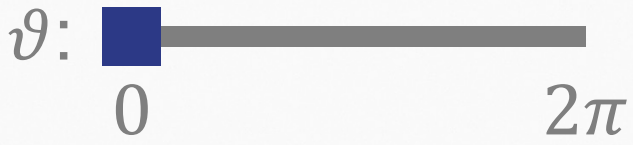
linear polarization (horizontal / vertical)

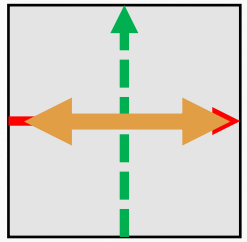
linear polarization (diagonal / antidiagonal)

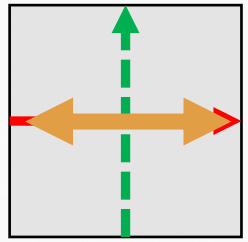
circular polarization



Twice rotation property



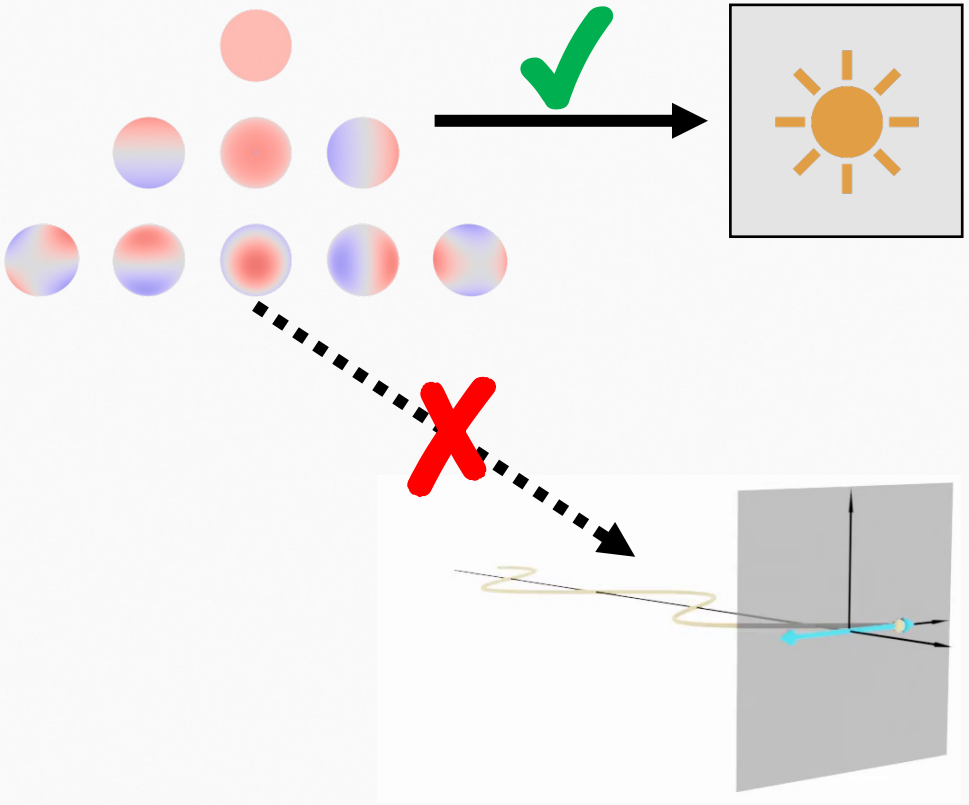




$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\vartheta & \sin 2\vartheta & 0 \\ 0 & -\sin 2\vartheta & \cos 2\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \cos 2\vartheta \\ -\sin 2\vartheta \\ 0 \end{bmatrix}$$

Problem: we need frames for each ray

Motivation



Spherical harmonics (SH) work well for scalar intensity

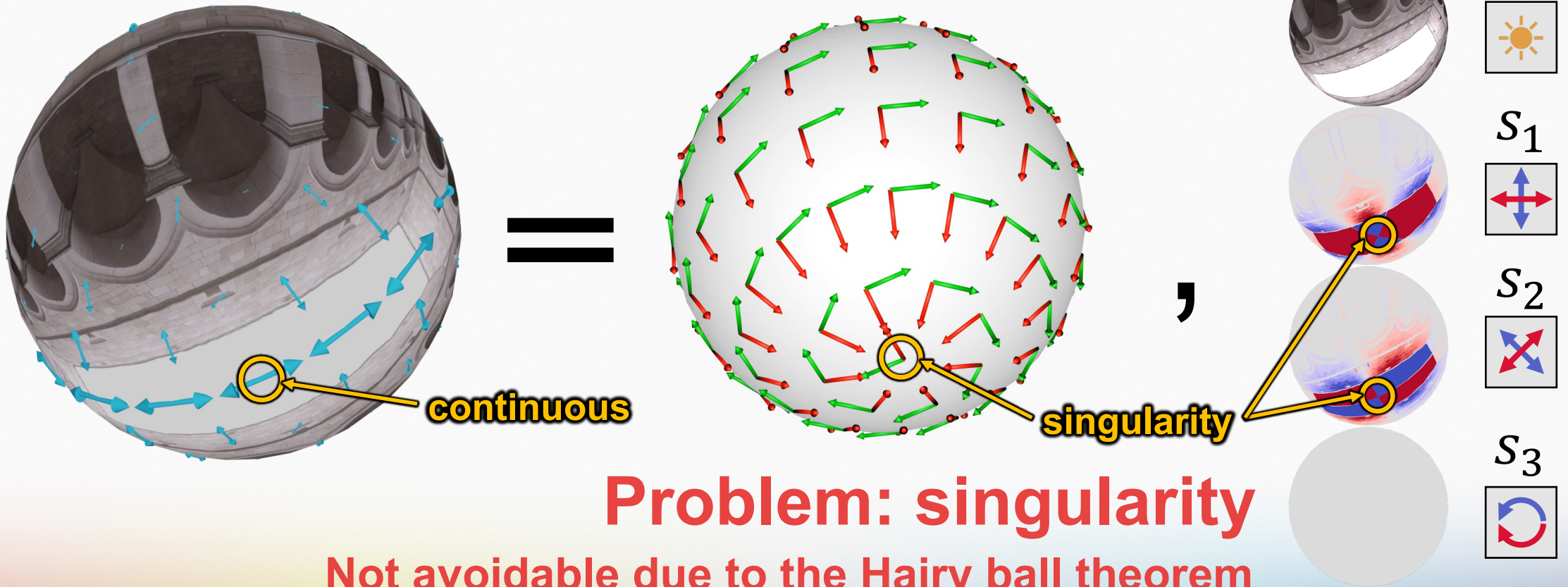
SH do not work for polarized light

The frame matters

Stokes vector field $\overrightarrow{f}$

Frame field $\vec{\mathbf{F}}(\hat{\omega})$

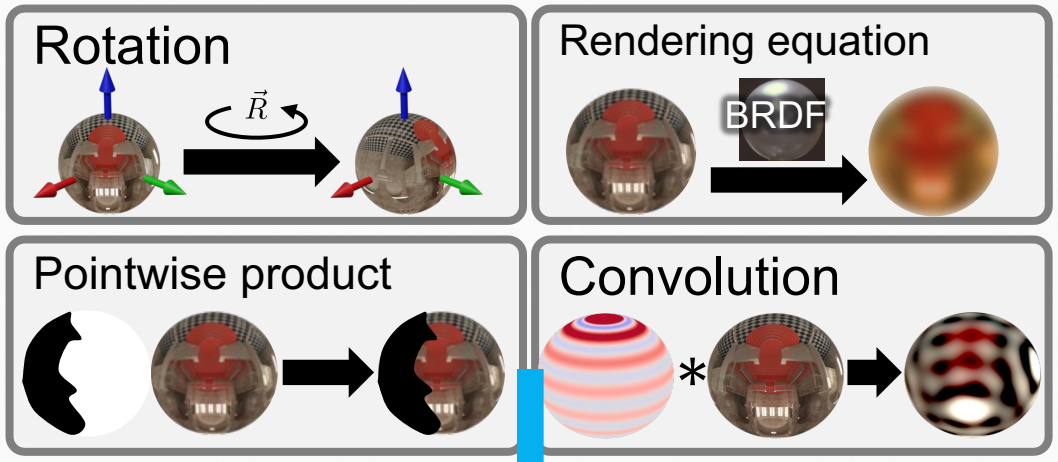
Stokes parameters w.r.t frame



Problem: singularity

Not avoidable due to the Hairy ball theorem

Why SH work well for scalar intensity?



Angular domain

Frequency domain

SH theory

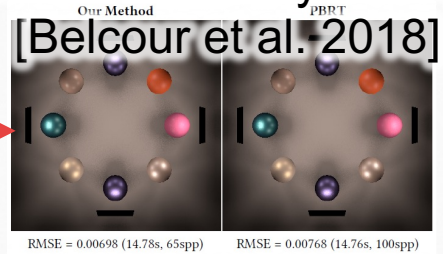
Frequency domain operations

Benefits:

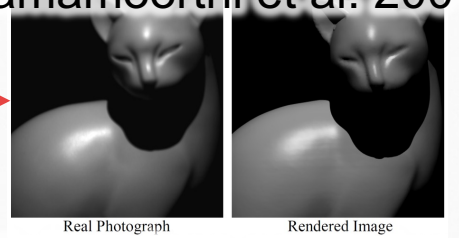
- ✓ **Rotation invariance**
- ✓ **Convolution to elementwise product**
- ✓ Other benefits…

Precomputed radiance transfer
[Sloan et al. 2002]

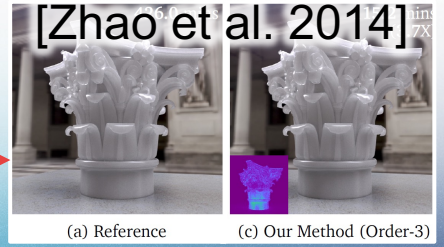
Monte Carlo ray tracing
[Belcour et al. 2018]



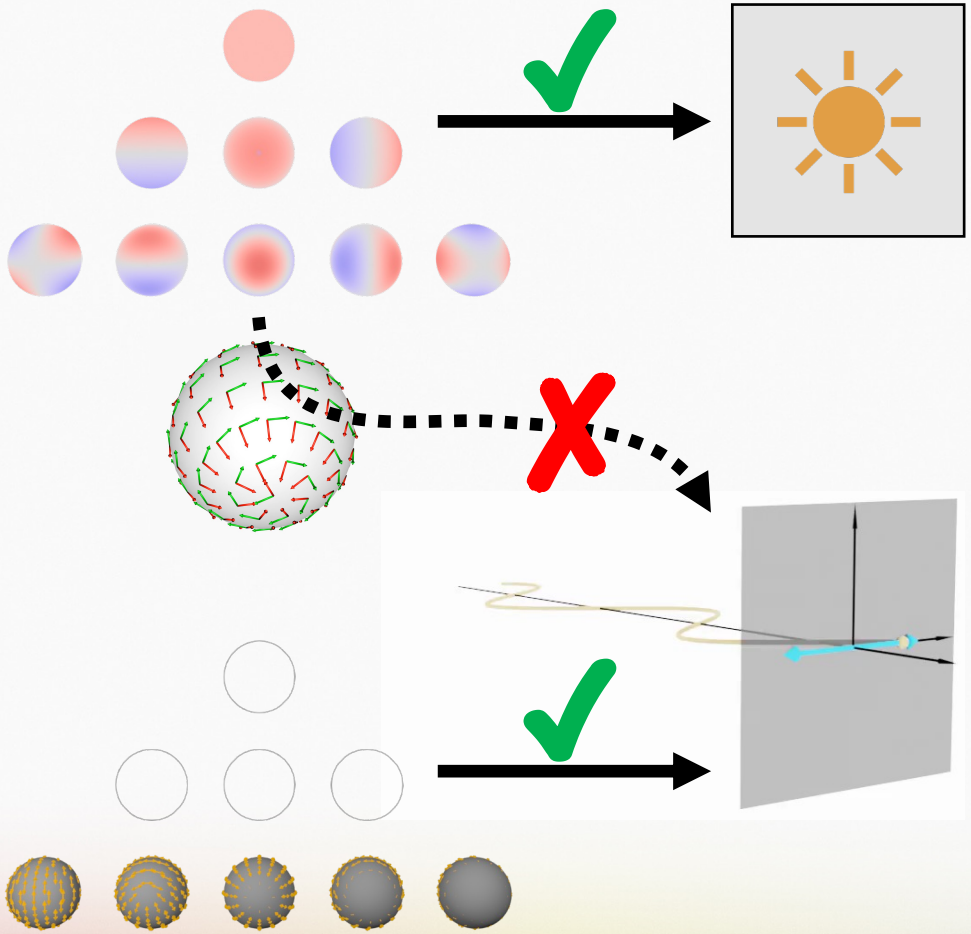
Appearance capture
[Ramamoorthi et al. 2001]



Subsurface scattering
[Zhao et al. 2014]



Overview



Why spherical harmonics (SH) work well for scalar intensity?

- ✓ Rotation invariance
- ✓ Convolution into elementwise product

SH do not work for polarized light

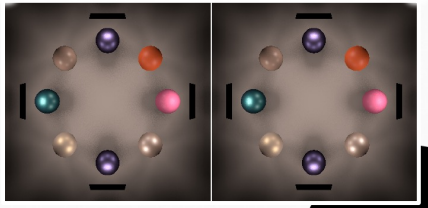
- ✗ No rotation invariance

Our polarized SH work for polarized light

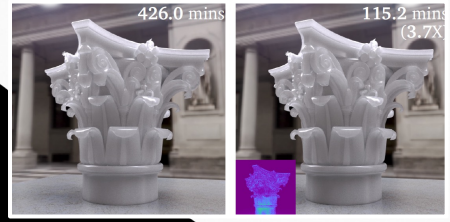
- ✓ Rotation invariance
- ✓ Convolution into nearly elementwise product

Contribution

Monte Carlo ray tracing
[Belcour et al. 2018]

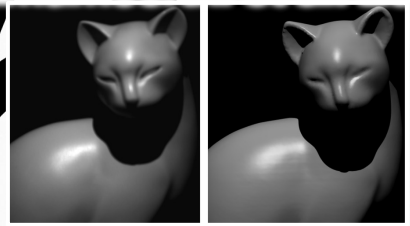


Subsurface scattering
[Zhao et al. 2014]

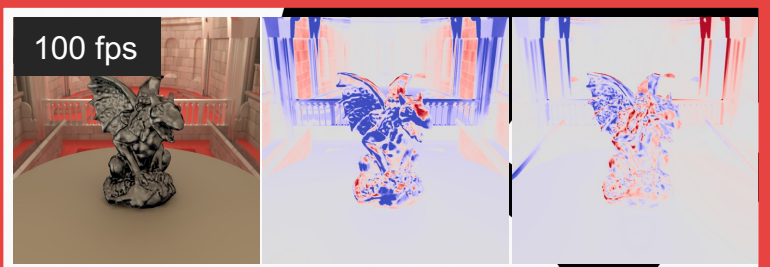


Precomputed radiance transfer
[Sloan et al. 2002]

Appearance capture
[Ramamoorthi et al. 2001]



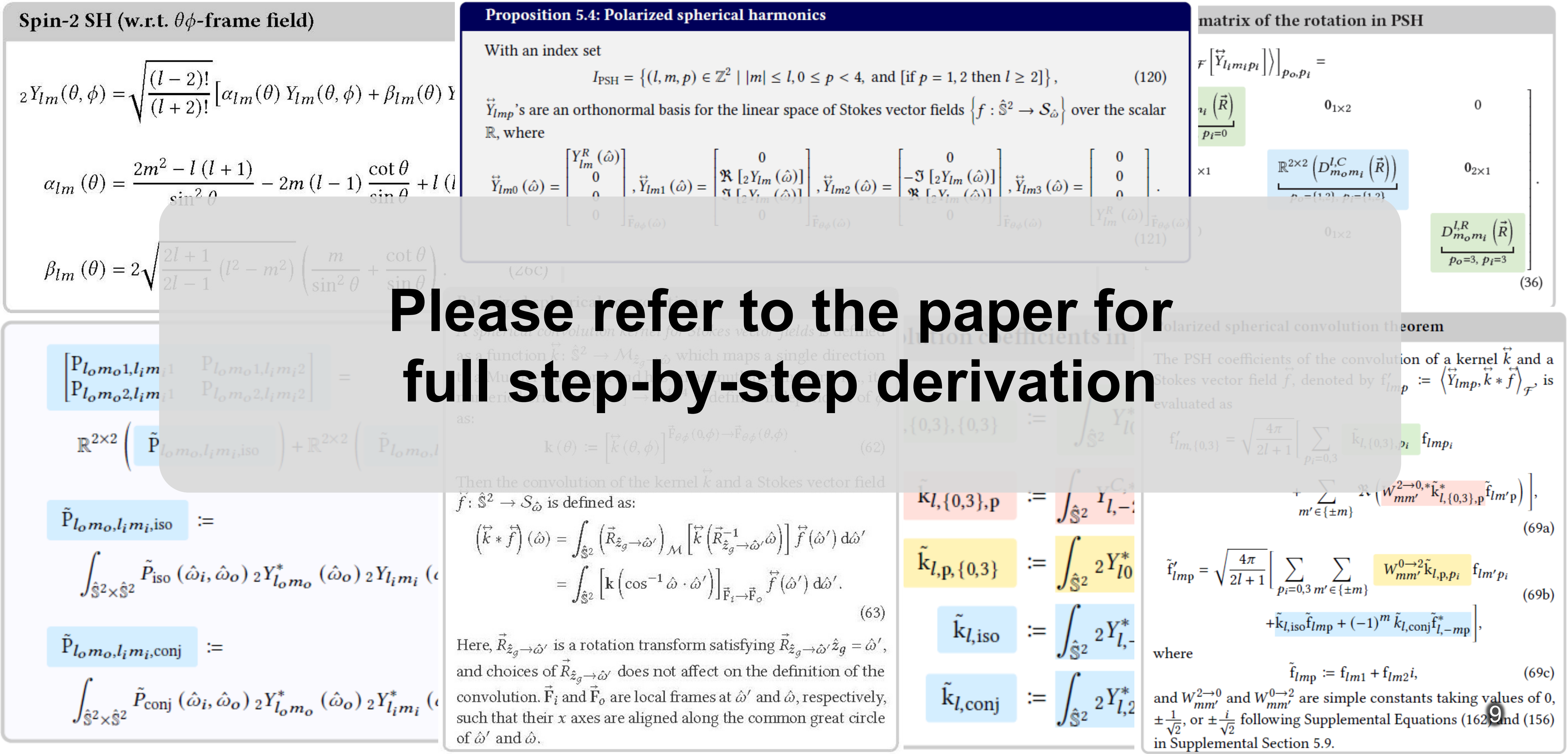
Precomputed polarized radiance transfer



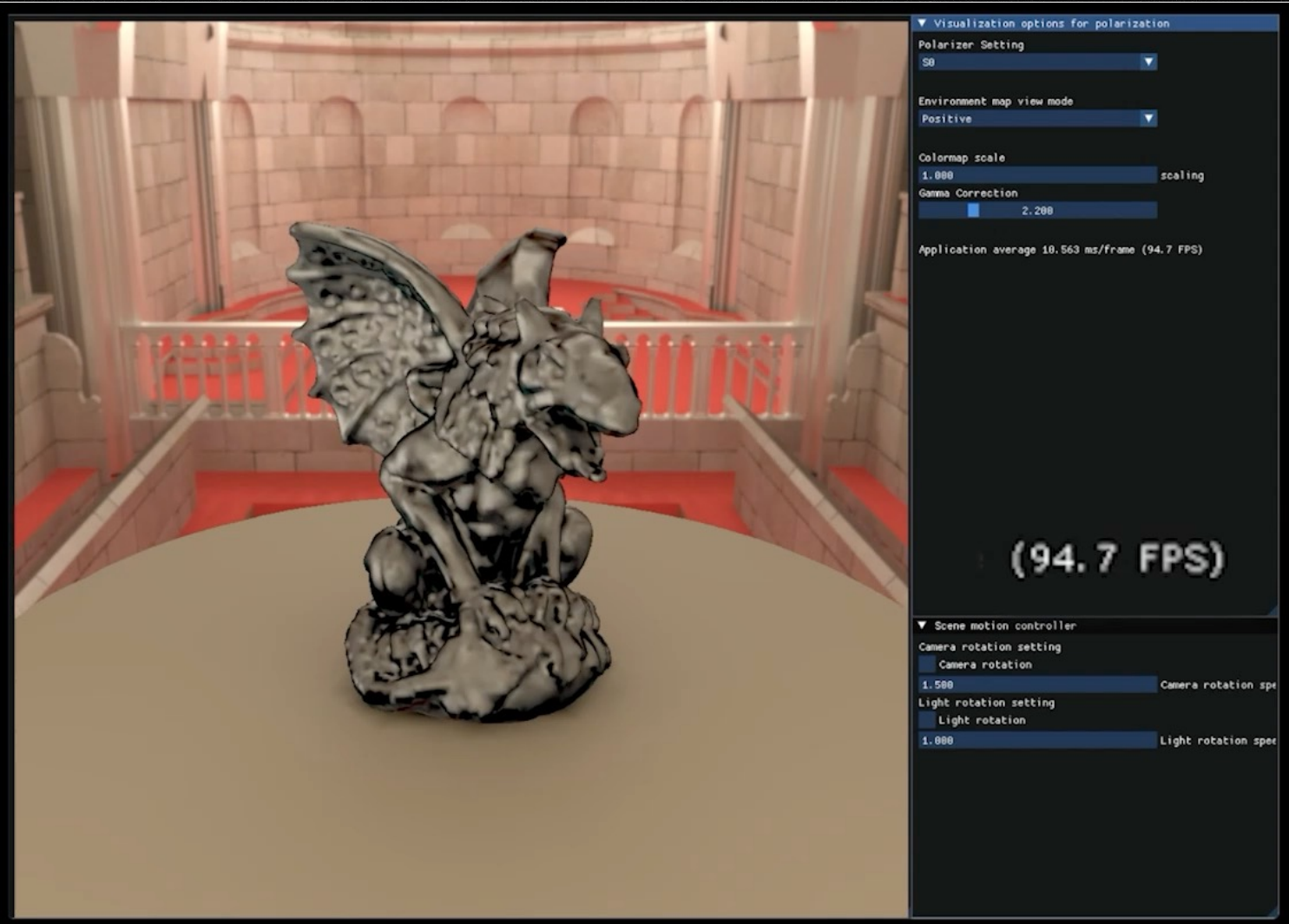
SH theory

Polarized SH theory

Polarized spherical harmonics theory



Real-time polarized rendering



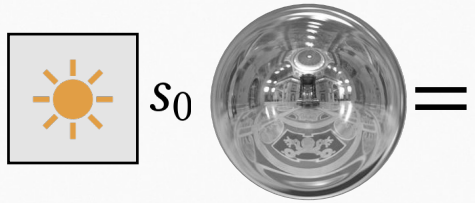
Intensity (s_0)

Rendered under a polarized environment map

Polarized spherical harmonics

Polarized SH

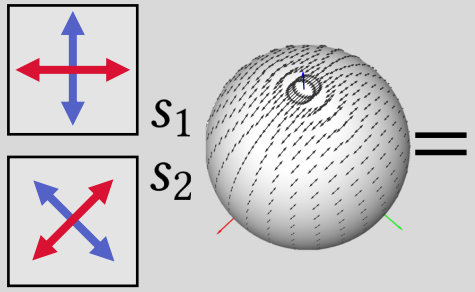
scalar (intensity)



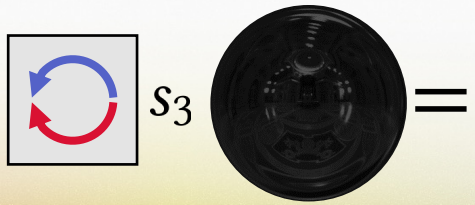
(conventional)

spin-0 SH

Linear polarization



scalar (circular polar.)



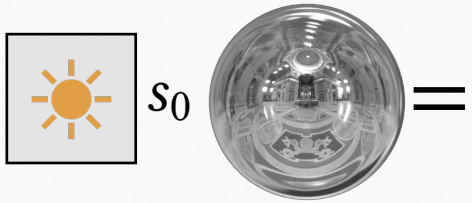
(conventional)

spin-0 SH

Polarized spherical harmonics

Polarized SH

scalar (intensity)

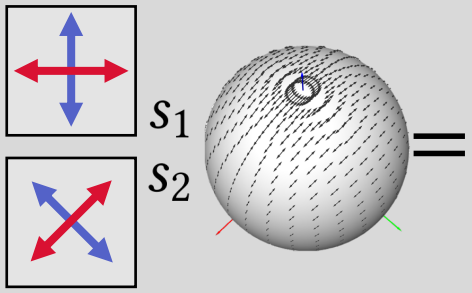


(conventional) spin-0 SH

f_{000} + $f_{1,-10}$ + f_{100} + f_{110}

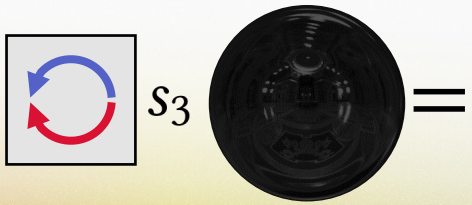
+$f_{2,-20}$ + $f_{2,-10}$ + f_{200} + f_{210} + f_{220}

Linear polarization



spin-2 SH

scalar (circular polar.)



(conventional) spin-0 SH

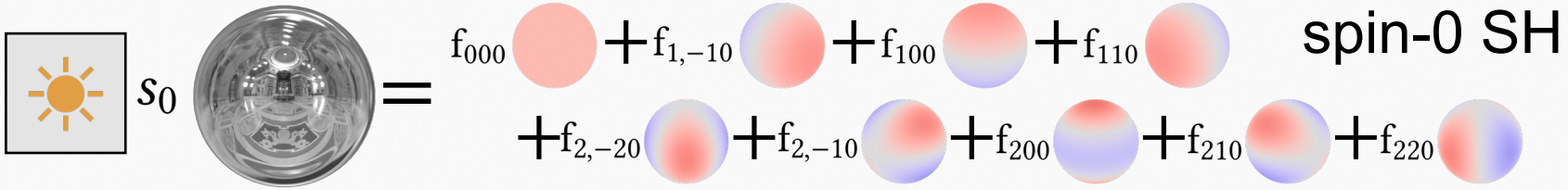
f_{003} + $f_{1,-13}$ + f_{103} + f_{113}

+$f_{2,-23}$ + $f_{2,-13}$ + f_{203} + f_{213} + f_{223}

Polarized spherical harmonics

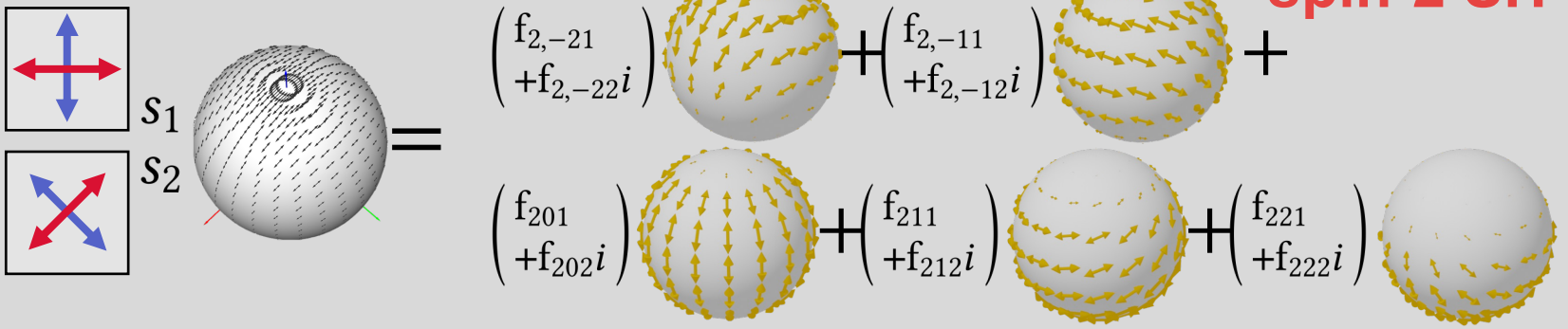
Polarized SH

scalar (intensity)



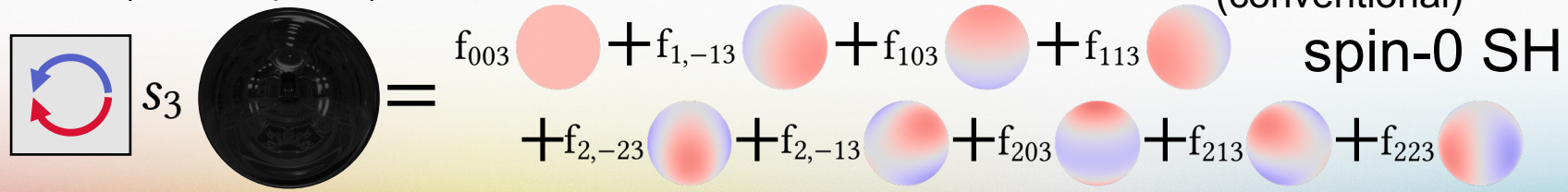
(conventional) spin-0 SH

Linear polarization



spin-2 SH

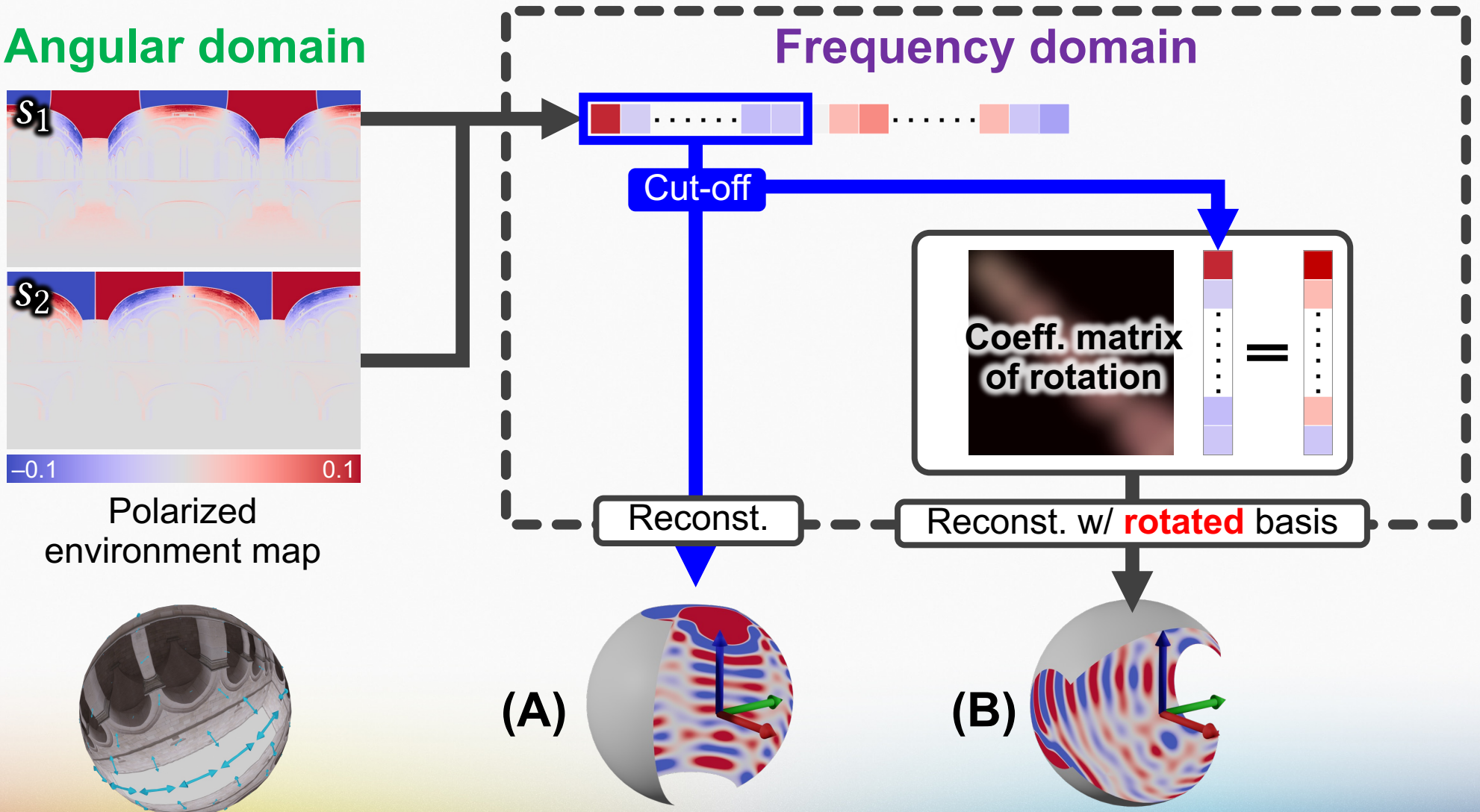
scalar (circular polar.)



(conventional) spin-0 SH

- ✓ **Rotation invariance**
- ✓ **Convolution to elementwise product**

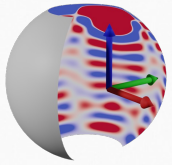
Rotation invariance for Stokes vectors



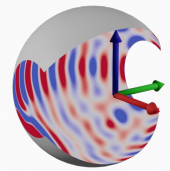
Rotation invariance for Stokes vectors

Coeff. matrix of rotation

(A)

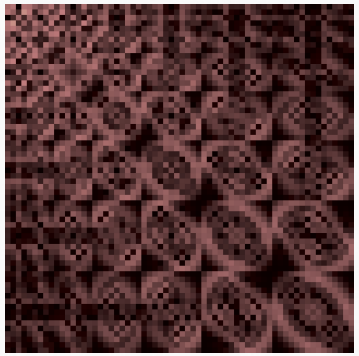


(B)



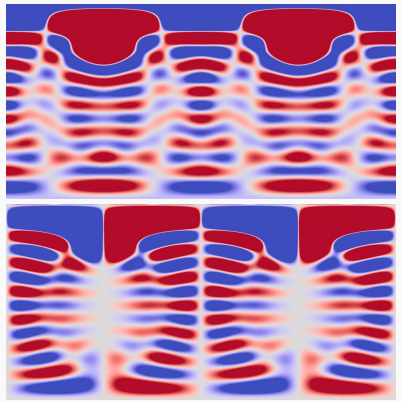
SH + frame field

- ✗ no rotation invariance
- ✗ offline
  (grid-based simulation)

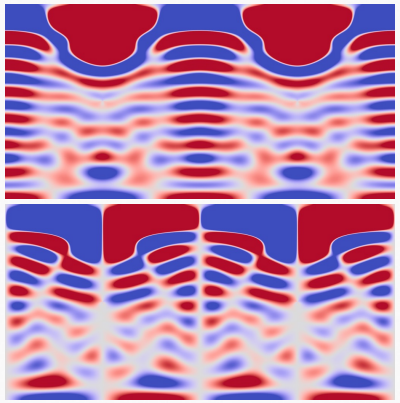


s_1

s_2

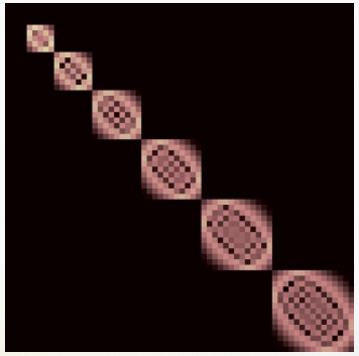


$\neq$



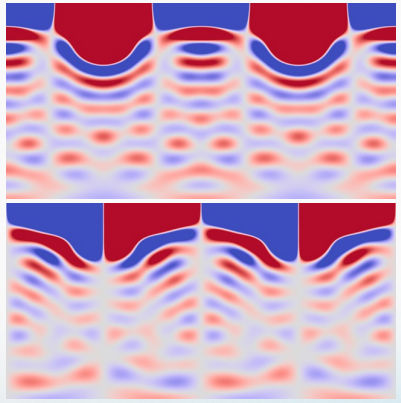
Spin-2 SH (Ours)

- ✓ rotation invariance
- ✓ real-time (analytic)

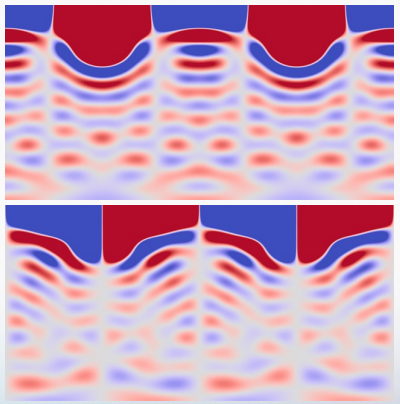


s_1

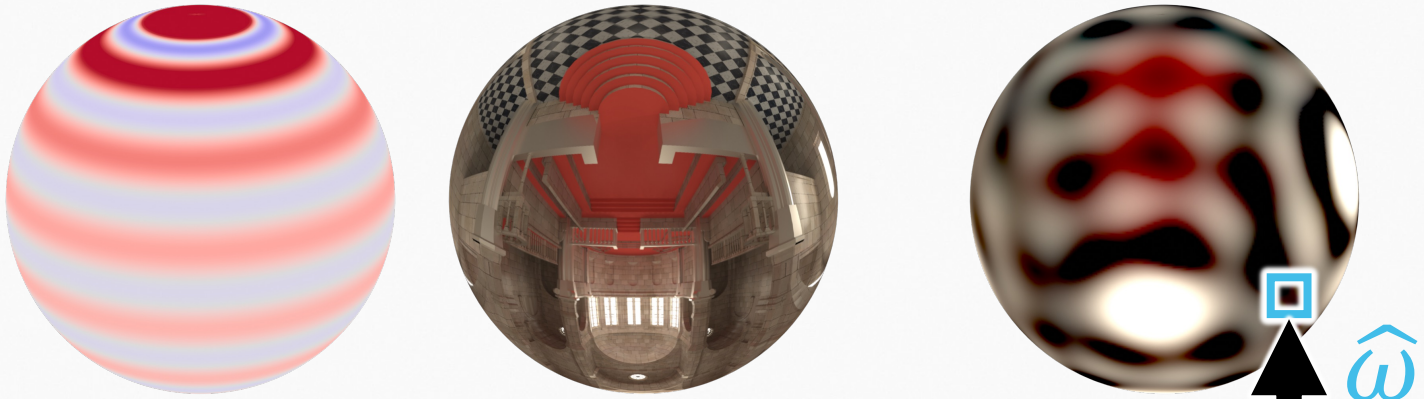
s_2



$=$



Scalar spherical convolution



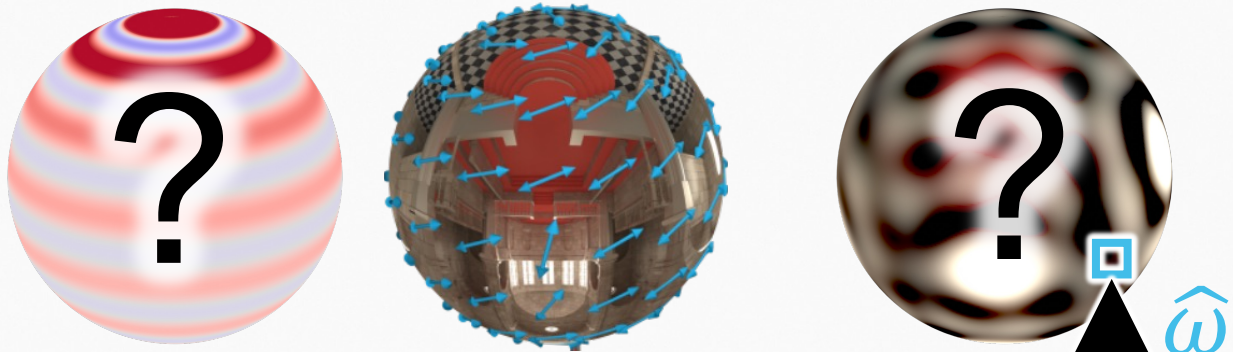
$$k \quad * \quad f \quad = \quad h$$

$\hat{\omega}$

$$k * f(\hat{\omega}) = \int \; \cdot \; \mathrm{d}\hat{\omega}'$$

$\hat{\omega}$

Polarized spherical convolution?



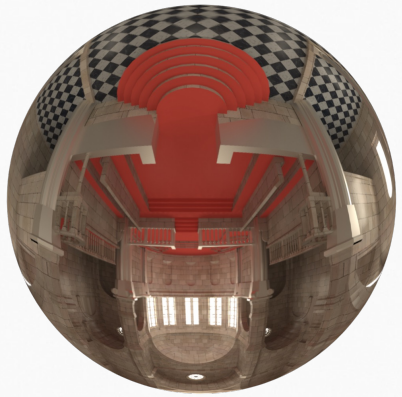
$$k * f(\hat{\omega}) = \int \; ? \; \cdot \; \mathrm{d}\hat{\omega}'$$

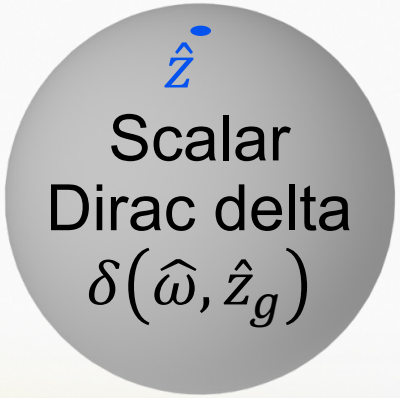
If f becomes a **Stokes vector field**….

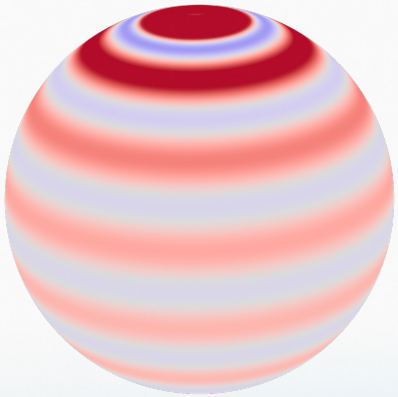
What should k become? Scalar? Stokes vector? Mueller matrix?

Scalar spherical convolution

= Rotation equivariant linear operator

$T[$  $] =$ 

$T[$ 

$] =$ 

kernel k

Polarized spherical convolution

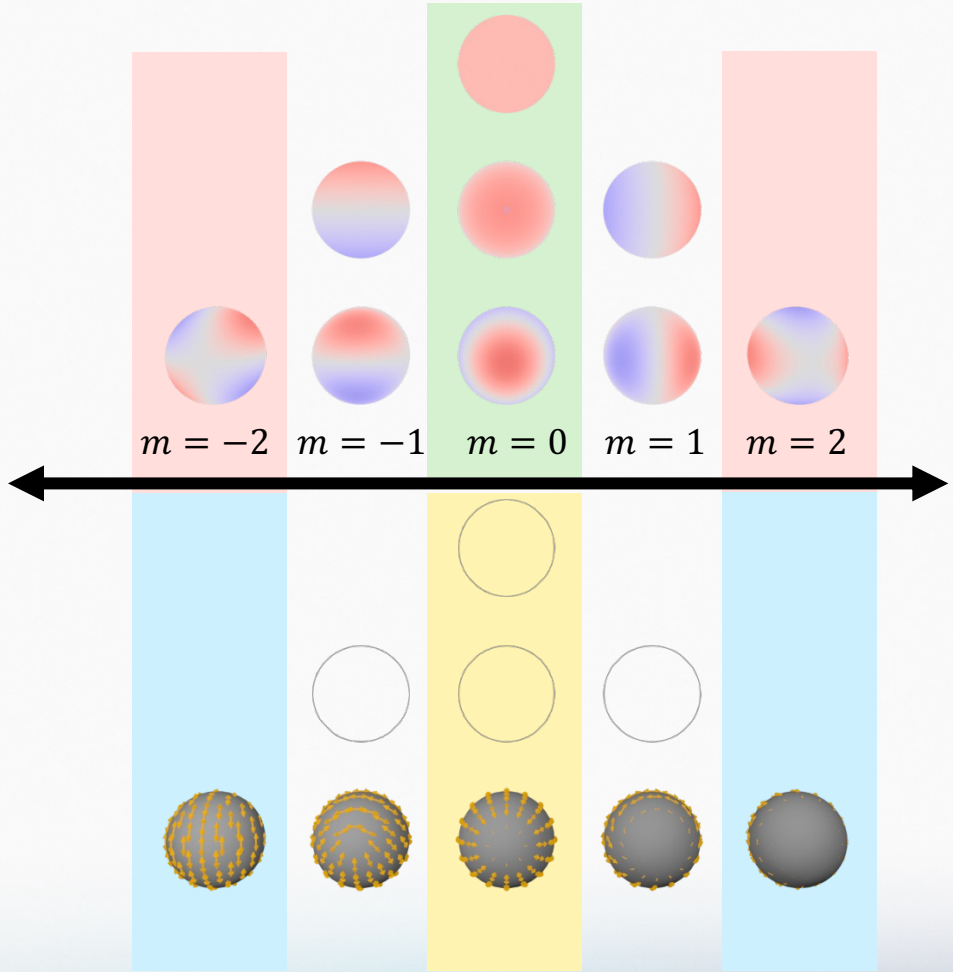
Angular domain

Frequency domain

Convolution kernel

$T\left[\right.$ Scalar Dirac delta $\delta(\hat{\omega}, \hat{z}_g)$ $\left.\right] =$ spin 0-to-0 $\oplus$ spin 0-to-2

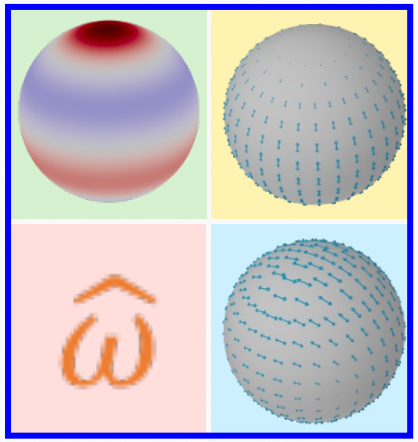
$T\left[\right.$ Spin-2 Stokes vector Dirac delta $[1]_{\vec{\mathbf{F}}_g}\delta(\hat{\omega}, \hat{z}_g)$ $\left.\right] =$ spin 2-to-0 $\oplus$ spin 2-to-2



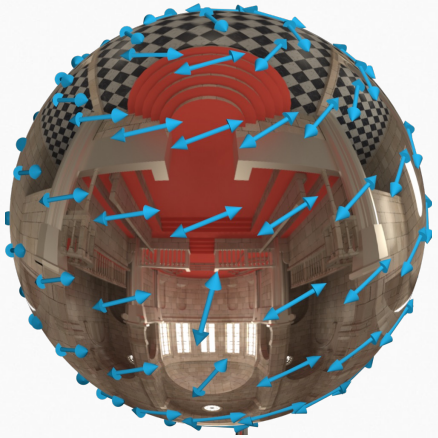
Kernels belong to only subspaces → Efficient elementwise product

Polarized spherical convolution

kernel k source f result $k * f$



$*$



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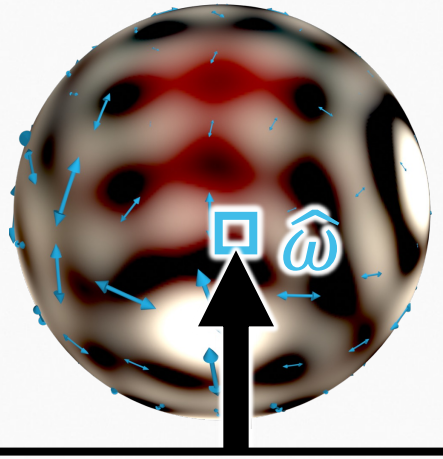
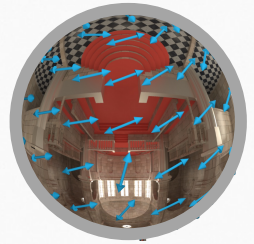


$$k * f(\hat{\omega}) = \int \begin{bmatrix} k_{00}(\theta) & k_{01}(\theta) & k_{02}(\theta) & k_{03}(\theta) \\ k_{10}(\theta) & k_{11}(\theta) & k_{12}(\theta) & k_{13}(\theta) \\ k_{20}(\theta) & k_{21}(\theta) & k_{22}(\theta) & k_{23}(\theta) \\ k_{30}(\theta) & k_{31}(\theta) & k_{32}(\theta) & k_{33}(\theta) \end{bmatrix} \text{ rotated by } \hat{\omega} \times$$



$$\mathrm{d}\hat{\omega}'$$

$$\theta = \cos^{-1} \hat{\omega} \cdot \hat{\omega}'$$

Angular domain

Frequency domain

Nearly elementwise product!

Real-time polarized rendering

111 fps

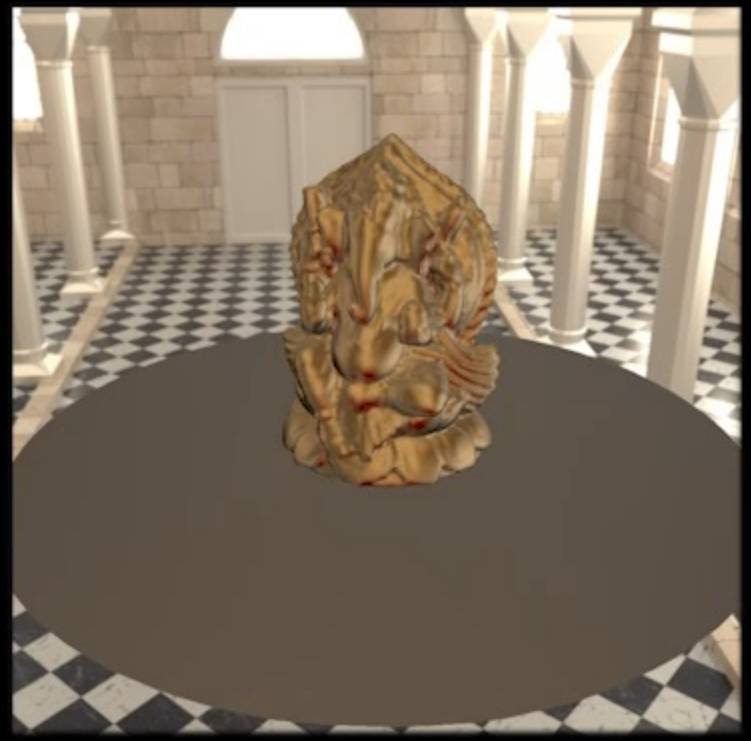
Intentisy (s_0)

Degree of Polarization

Angle of Linear Polarization

Rendered under a polarized environment map

Ablation: Frequency domain operations



Low order

Rotation & Rendering equation

+ Shadow

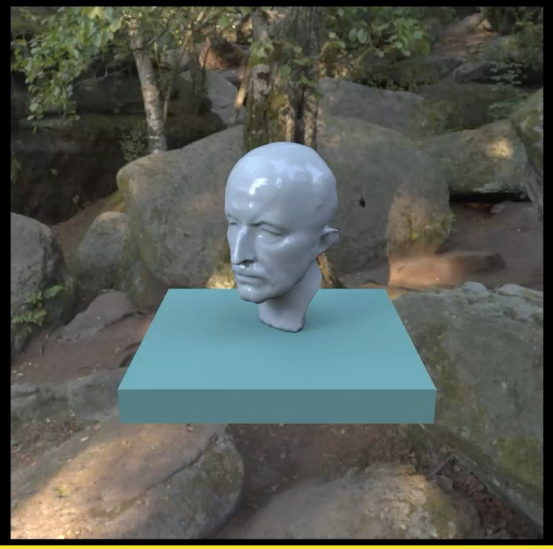
(Pointwise product)

s_0

+ High order

(Convolution)

Efficiency of convolution approximation



offline (Mitsuba3)

Ground truth

480 fps

$l \leq 4$

210 fps

$l \leq 5$

308 fps

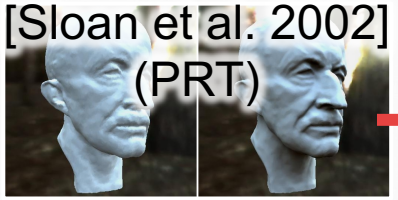
$l \leq 4$ convolution approx. for $5 \leq l \leq 9$

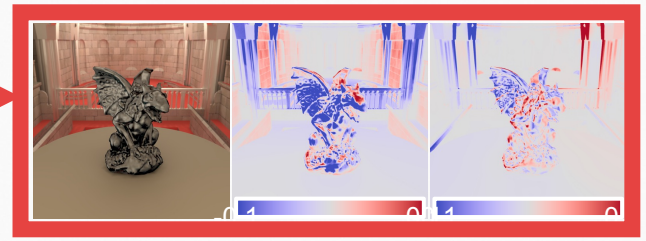
Intensity (s_0)

Conclusion and future work

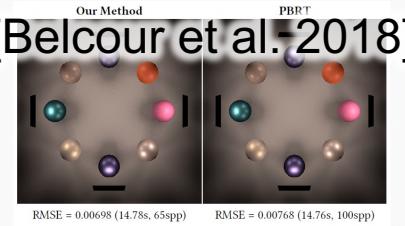
Precomputed polarized radiance transfer

[Sloan et al. 2002] (PRT)



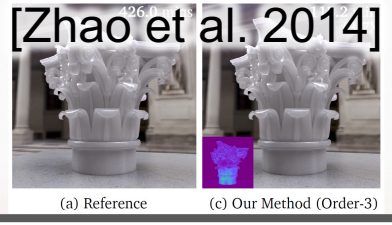


[Belcour et al. 2018]



[Ramamoorthi et al. 2002]

[Zhao et al. 2014]



SH theory

future work

Polarized SH theory

Future work

- Extend more techniques for SH to PSH
  - Polygonal area [Wang'18]
  - Fast triple product [Xin'21]
  - Removing ringing artifact [Sloan' 17]
- More applications
  - Polarized radiance fields (NeRF-like)
  - Spherical CNN for polarized light

THANK YOU

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