Birefractive Stereo Imaging for Single-Shot Depth Acquisition

Supplemental Material #1

Seung-Hwan Baek† Diego Gutierrez* Min H. Kim†

†KAIST *Universidad de Zaragoza, I3A

1 Calculation of E-Ray Direction Cosines

While we formulate all the light transport using global coordinates to estimate depth, Liang's formulae [1990] take local coordinates with respect to the plane of incidence at the incident point as origin. Therefore we need to convert from global to local coordinates before applying Liang's formulae. First, we build the basis rotation matrix M from local to global coordinates. M can be obtained from the cross products of the surface normal $\mathbf{n}$ and the unit vector of the incident ray $\hat{\mathbf{l}}^i$ as follows: $M = [\mathbf{n}, \mathbf{n} \times \hat{\mathbf{l}}^i, \mathbf{n} \times (\mathbf{n} \times \hat{\mathbf{l}}^i)]^\intercal$. Second, the inverse of M converts the global frame $\hat{\mathbf{l}}^i$ to the local frame $\hat{\mathbf{l}}^i_L$: $\hat{\mathbf{l}}^i_L = M^{-1}\hat{\mathbf{l}}^i$. Finally, we calculate the e-ray unit vector $\hat{\mathbf{l}}^r_{eL}$ in local coordinates from the incident ray vector $\hat{\mathbf{l}}^i_L$ and the optical axis $\mathbf{a} = [a_x, a_y, a_z]^\intercal$ [Liang 1990].

Once we convert the incident ray vector, we calculate the direction cosines of the e-ray $\hat{\mathbf{l}}^i_{eL}$ as follows: First, the incident angle θ can be obtained from the normalized dot product of $\mathbf{n} \cdot \hat{\mathbf{l}}^i$. We define the intermediate angles θ_2 and θ_3 as the angle between the e-ray and the local x-axis, and the angle between the e-ray and $\mathbf{a}$ and α (the dispersion angle of the e-ray), respectively, with the refractive indices of o-/e-rays (η_o and η_e):

$$\begin{aligned}
\cot\theta_2 &= \frac{2a_x a_y\left(\eta_o^2-\eta_e^2\right)}{2[\eta_o^2+a_x^2(\eta_e^2-\eta_o^2)]} \pm \\
&\quad \frac{2\eta_o\sqrt{\frac{\eta_o^2 n_e^2+\eta_e^2 a_x^2(\eta_e^2-\eta_o^2)}{\sin^2\theta} - \left[\eta_o^2+(\eta_e^2-\eta_o^2)\left(a_x^2+a_y^2\right)\right]}}{2[\eta_o^2+a_x^2(\eta_e^2-\eta_o^2)]}, \\
\cos\theta_3 &= a_x\cos\theta_2 + a_y\sin\theta_2, \\
\tan\alpha &= \frac{\left(\eta_e^2-\eta_o^2\right)\tan\theta_3}{\eta_e^2+\eta_o^2\tan^2\theta_3}.
\end{aligned}$$

Then, the direct cosines of the e-ray unit vector $\hat{\mathbf{l}}^r_{eL}$ can be calculated as follows:

$$\begin{aligned}
\hat{l}^r_{eL,x} &= \cos\alpha\cos\theta_2 + \frac{\sin\theta_2\sin\alpha(a_x\sin\theta_2 - a_y\cos\theta_2)}{\sqrt{a_z^2+(a_x\sin\theta_2-a_y\cos\theta_2)^2}}, \\
\hat{l}^r_{eL,y} &= \cos\alpha\sin\theta_2 - \frac{\cos\theta_2\sin\alpha(a_x\sin\theta_2 - a_y\cos\theta_2)}{\sqrt{a_z^2+(a_x\sin\theta_2-a_y\cos\theta_2)^2}}, \\
\hat{l}^r_{eL,z} &= \frac{a_z\sin\alpha}{\sqrt{a_z^2+(a_x\sin\theta_2-a_y\cos\theta_2)^2}}.
\end{aligned}$$

We calculate the e-ray path from the local coordinates $\hat{\mathbf{l}}^r_{eL}$, by taking the thickness t of the crystal slab into account:

$$\mathbf{l}^r_{eL} = \frac{t}{\hat{l}^r_{eL,x}}\hat{\mathbf{l}}^r_{eL} = \begin{bmatrix} t & \frac{t}{\hat{l}^r_{eL,x}}\hat{l}^r_{eL,y} & \frac{t}{\hat{l}^r_{eL,x}}\hat{l}^r_{eL,z} \end{bmatrix}^\intercal.$$

Now, we convert the e-ray path vector in local coordinates to global coordinates by applying the transformation matrix M to the unit vector: $\mathbf{l}^r_e = M\left(\frac{t}{\hat{l}^r_{eL,x}}\mathbf{l}^r_{eL}\right)$. Calculating the e-ray vector $\mathbf{l}^r_e$ enables us to complete the e-ray transport in our system, by providing the deviation angle of the e-ray from the plane of incidence, and the length of the e-ray inside the birefringent medium.

References

Liang, Q.-T. 1990. Simple ray tracing formulas for uniaxial optical crystals. *Applied Optics 29*, 7, 1008–1010.