CS482: Interactive Computer Graphics

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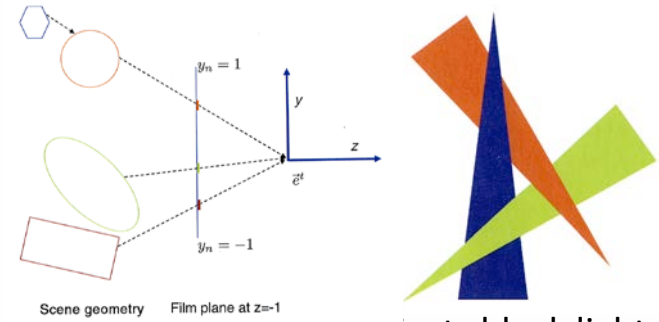
Chapter 11

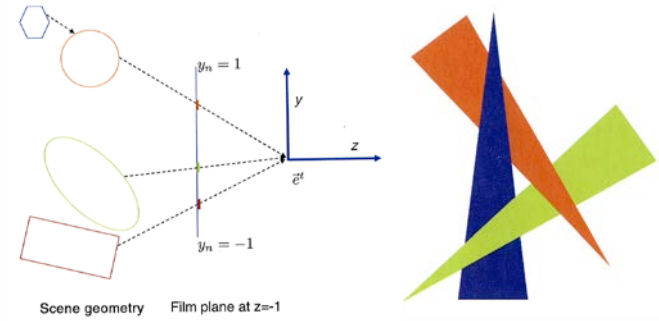
DEPTH

Visibility



- In the real world, opaque objects block light.
- We need to model this computationally.
- One idea is to render back to front and use overwriting
  - This will have problem with visibility cycles.

Visibility



- We could explicitly store everything hit along a ray and then compute the closest.
  - Make sense in a ray tracing setting, where we are working one pixel per ray at time, but not for OpenGL, where we are working one triangle at a time.

Z-buffer

- We will use z-buffer
- Triangle are drawn in any order
- Each pixel in frame buffer stores 'depth' value of closest geometry observed so far.
- When a new triangle tries to set the color of a pixel, we first compare its depth to the value stored in the z-buffer.
- Only if the observed point in this triangle is closer, we overwrite the color and depth values of this pixel.

Z-buffer

- This is done per-pixel, so there is no cycle problems.
- There are optimizations, where z-testing is done, before the fragment shading is done.

Other uses of visibility calculations

- Visibility to a light source is useful for shadows.
  - We will talk about shadow mapping later.
  - We will do shadow calculations in a ray tracer.
- Visibility computation can also be used to speed up the rendering process.
  - If we know that some object is occluded from the camera, then we don't have to render the object in the first place.
  - We can use a conservative test.

Basic mathematical model

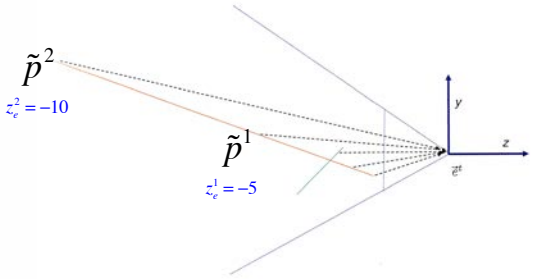
- For every point, we define its $[x_n, y_n, z_n]^t$ coordinates, using the following matrix expression:

$$\begin{bmatrix} x_n w_n \\ y_n w_n \\ z_n w_n \\ w_n \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} s_x & 0 & -c_x & 0 \\ 0 & s_y & -c_y & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$

- We now also have the value $z_n = \frac{-1}{z_e}$
- Our plan is to use this z_n value to do depth comparison in our z-buffer.

Correct ordering

- Given two points $\tilde{p}^1$ and $\tilde{p}^2$ with eye coordinates $[x_e^1, y_e^1, z_e^1, 1]^t$ and $[x_e^2, y_e^2, z_e^2, 1]^t$.
- Suppose that they both are in front of the eye, i.e., $z_e^1 < 0$ and $z_e^2 < 0$.
- And suppose that $\tilde{p}^1$ is closer to the eye than $\tilde{p}^2$, that is $z_e^2 < z_e^1$
- Then $-\frac{1}{z_e^2} < -\frac{1}{z_e^1}$, meaning $z_e^2 < z_e^1$

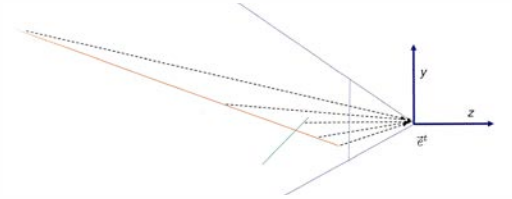


Projective transform

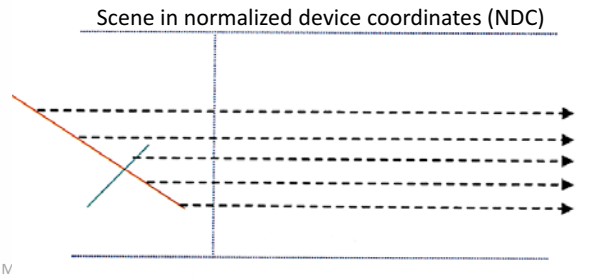
- We can now think of the process of taking points (given by eye coordinates) to points (given by normalized device coordinates) as an honest-to-goodness 3D geometric transformation.
- This kind of transformation is generally neither linear nor affine, but is something called a *3D projective transformation*.
- Projective transformation preserve co-linearity and co-planarity of points.

Co-planarity of points

- Eye space

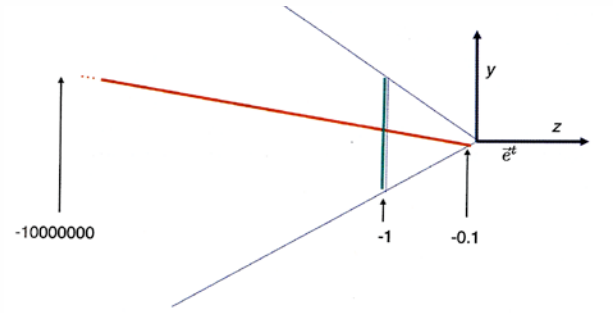


- NDC



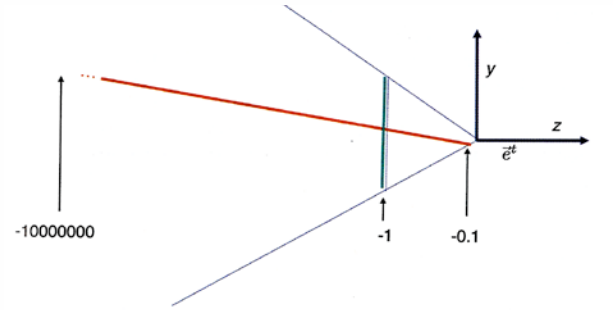
z_n interpolation is right

- Preservation of coplanarity: for points on a fixed triangle, we will have $z_n = ax_n + by_n + c$, for some fixed a, b and c
- Thus, the correct z_n value for a point can be computed using linear interpolation over the 2D image domain as long as we know its value at the three vertices of the triangle.



z_e interpolation is wrong

- Linear interpolation of z_e values over the screen would produce wrong answer.
- All the blue pixels would have interpolated z_e value of -1. On the orange triangle value would become less than -1 almost immediately.
- The entire image would become blue!!!



Numerics

- There can be numerical difficulties when computing z_n. As z_e goes towards zero, $z_n = \frac{-1}{z_e}$ the z_n value diverges off towards infinity.
- Conversely, points very far from the eye have z_n values very close to zero. The z_n of two such far away points may be indistinguishable in a finite precision representation, and thus the z-buffer will be ineffective in distinguishing which is closer to the eye.

Solution: near/far

- Solution: replacing the third row of the matrix with more general row $\begin{bmatrix} 0 & 0 & \alpha & \beta \end{bmatrix}$

$$\rightarrow \begin{bmatrix} x_n w_n \\ y_n w_n \\ z_n w_n \\ w_n \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} s_x & 0 & -c_x & 0 \\ 0 & s_y & -c_y & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$

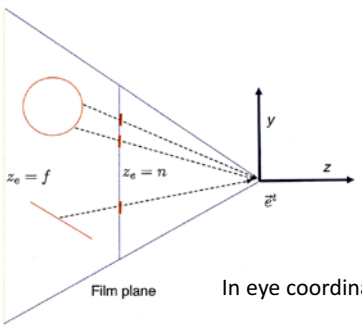
- It is easy to verify that if the value α and β are both positive, then the z-ordering of points (assuming they all have negative z_e values) is preserved under the projective transform.

Solution: near/far

- To set α and β, we first select depth value n and f called the *near* and *far* values (both negative), such that our main region of interest in the scene is sandwiched between $z_e = n$ and $z_e = f$
- Given these selections we set

$$\alpha = \frac{f+n}{f-n}$$

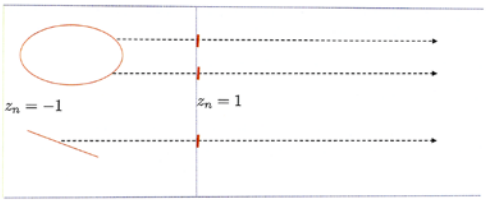
$$\beta = -\frac{2fn}{f-n}$$



In eye coordinates

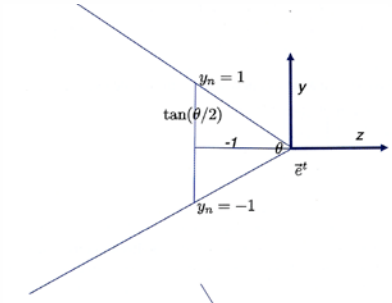
Solution: near/far

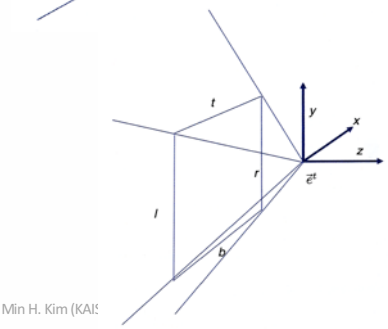
- We can verify now that any point with $z_e = f$ maps to a point with $z_n = -1$ and that a point with $z_e = n$ maps to a point with $z_n = 1$
- Any geometry not in this [near…far] range is clipped away by OpenGL and ignored.



In normalized device coordinates

Proj. Trans.: Eye coor. → NDC





$$\begin{bmatrix} \frac{1}{\alpha \tan\left(\frac{\theta}{2}\right)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan\left(\frac{\theta}{2}\right)} & 0 & 0 \\ 0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & -\frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

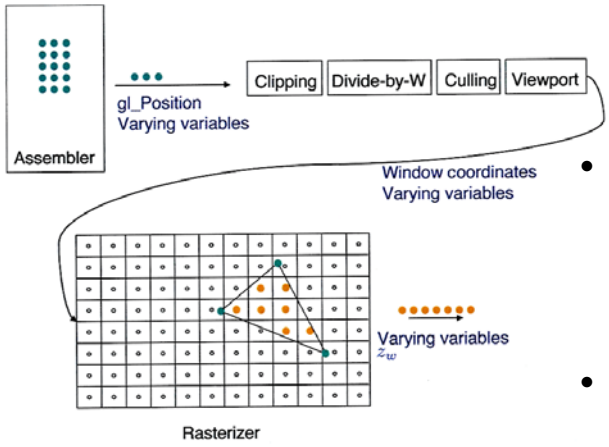
Codes

- In OpenGL, use the z-buffer is turned on with a call to glEnable(GL_DEPTH_TEST).
- We also need a call to glDepthFunc(GL_GREATER), since we are using a right handed coordinate system where 'more-negative' is 'farther from the eye'.
- In real life, you may see other conventions (for how to interpret n and f, some of the signs of the matrix, and the handedness of the ultimate z-test.

Chapter 12

RASTERIZATION

Path from vertex to pixel



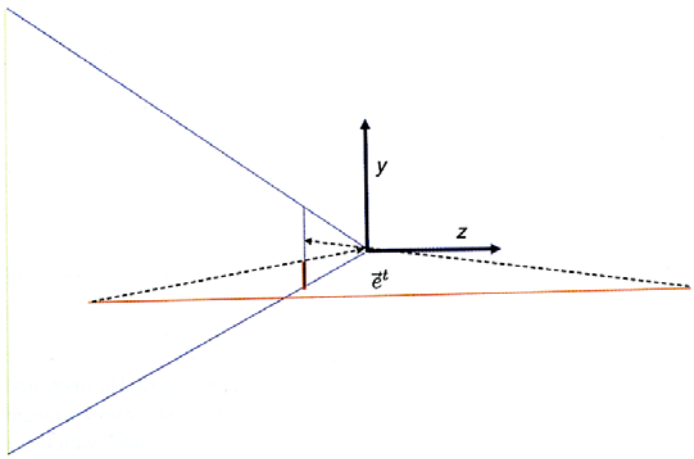
- Three vertices have passed through the vertex shader
- Follow it journey to be a bunch of pixels

Varying variables

- Varying variables provides an interface between the vertex and fragment shader.
- When the primitives are assembled and fragments computed, for each fragment there is a set of variables that are interpolated automatically and provided to the fragment shader.
- An example is the color of the fragment. The color that arrives at the fragment shader is the result of the interpolation of the colors of the vertices that make up the primitive.
  - varying float intensity;

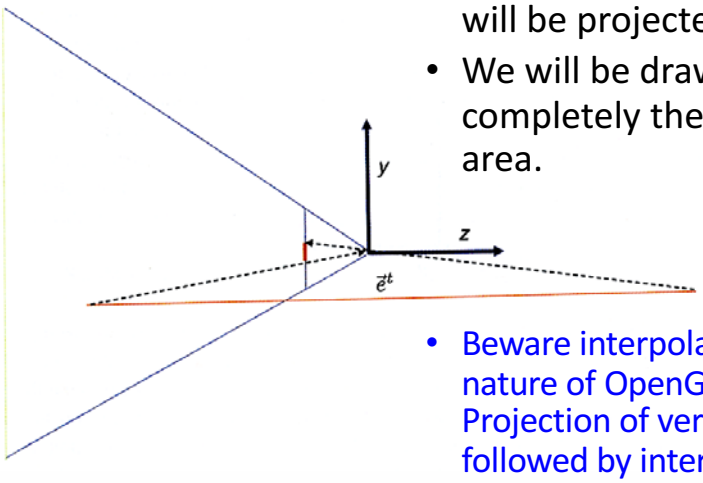
Clipping

- What if there is a vertex behind you?



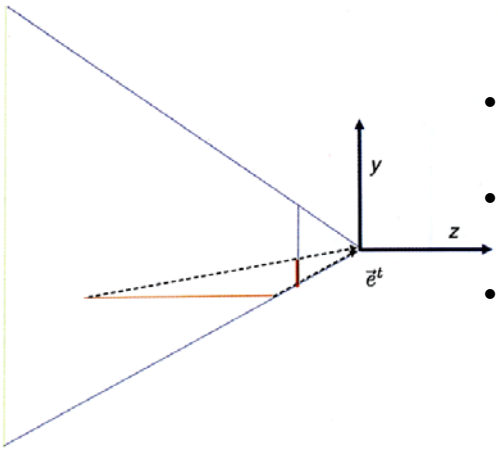
Clipping

- Not only front vertex but also back vertex will be projected
- We will be drawing in completely the wrong area.



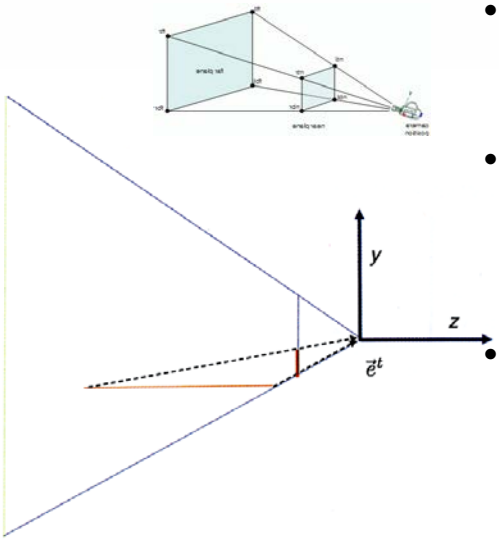
- Beware interpolation nature of OpenGL: Projection of vertices followed by interpolation

Clipping



- Processing for triangles that are fully or partially out of view
- We don't want to see behind us
- We want to minimize processing
- The tricky part will be to deal with eye-spanning triangles.

Eye spanners

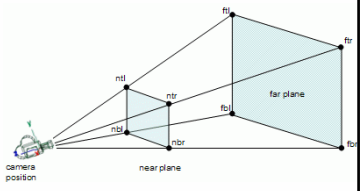


- Back vertex projects higher up in the image
- Filling in the in-between pixels will fill in the wrong region.
- Solution: slice up the geometry by the six faces of the view frustum

Clipping coordinates

- Eye coordinates (projected) → clip coordinates → normalized device coordinates (NDCs)
- (reminder) Dividing clip coordinates (x_c, y_c, z_c, w_c) by the $w_c (w_c = w_n)$ component (the fourth component in the homogeneous coordinates) yields normalized device coordinates (NDCs).

$$\begin{bmatrix} x_n w_n \\ y_n w_n \\ z_n w_n \\ w_n \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} s_x & 0 & -c_x & 0 \\ 0 & s_y & -c_y & 0 \\ 0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix}$$



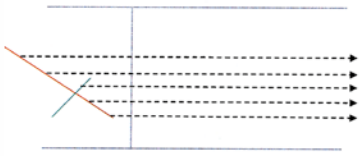
Clipping coordinates

- Suppose that x_c / w_c is the same as $-x_c / -w_c$. What if w_c is zero?
- If you wait for normalized device coordinates (NDCs) where the vertex has flipped, and it's too late to do the clipping.
- We could do in the eye space, but then would need to use the camera parameters
- The solution is to use clip coordinates: post-matrix-multiply but pre-divide.
- No divide = no flipping

Clipping coordinates

- Recall that we want points in the range:

$$-1 < x_n < 1$$
$$-1 < y_n < 1$$
$$-1 < z_n < 1$$

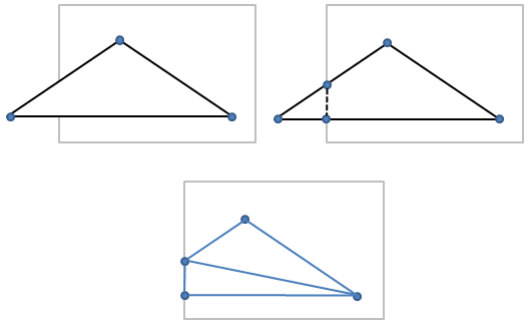


- In clip coordinates this is:

$$\begin{bmatrix} x_n w_n \\ y_n w_n \\ z_n w_n \\ w_n \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} \qquad \begin{aligned} -w_c < x_c < w_c \\ -w_c < y_c < w_c \\ -w_c < z_c < w_c \end{aligned}$$

Clipping coordinates

- Primitives totally inside the clipping volume are not altered. Primitives outside the viewing volume are discarded. Primitives whose edges intersect the boundaries of the clipping volume are clipped (learn later).



Clipping coordinates → NDCs

- Clipping is done, we can now divide by w_c (typically $-z_e$) to obtain normalize device coordinates.

$$\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} x_c / w_c \\ y_c / w_c \\ z_c / w_c \\ w_c / w_c \end{bmatrix}$$