


CS380: Introduction to Computer Graphics  
Geometric Modeling  
Chapter 23

Min H. Kim  
KAIST School of Computing


Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012*



## Announcement

- Final Exam:
  - Roughly after the mid-term exam
- Thanks for this semester!


Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012*



Light Transport

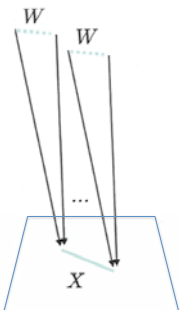
## SUMMARY

Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012*



## Radiant flux

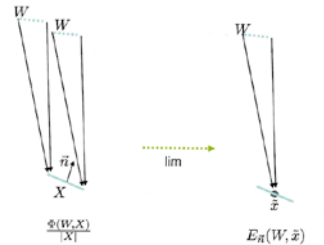
- A bunch of photons flying through space in a direction.
- We have a sensor  $(W, X)$  out in space
  - $W$  is a wedge of directions
  - Pass through the surface  $X$
- Each photon carries energy in units of *joules*
- Radiant flux: energy measured in seconds, measured in *watts*:  $\Phi(W, X)$



Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012*

## Irradiance

- Measured radiant flux by the area of the sensor (measured in square meters)  $E(W, X) := \frac{\Phi(W, X)}{|X|}$
- Smaller and smaller  $X$  around a single point  $\tilde{x}$  (under continuity assumptions about  $\Phi$ )  $\rightarrow$  irradiance  $E_{\tilde{n}}(W, \tilde{x})$
- Note  $E$  at a different normal  $\tilde{n}'$  is different  $E_{\tilde{n}}(W, \tilde{x}) \neq E_{\tilde{n}'}(W, \tilde{x})$



Min H. Kim (KAIST) Foundations of 3D Computer C

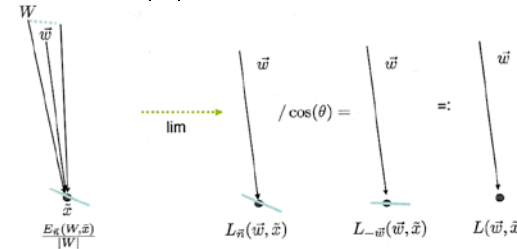
5

## Radiance

- In  $E_{\tilde{n}}(W, \tilde{x})$ , shrink  $W$  down to a single vector, dividing out by  $|W|$ , the solid angle measure of  $W$  (unit: steradians)  $\rightarrow$  radiance

$$L_{\tilde{n}}(W, \tilde{x}) = \frac{E_{\tilde{n}}(W, \tilde{x})}{|W|}$$

(the wedge of all direction covers  $4\pi$  steradians)

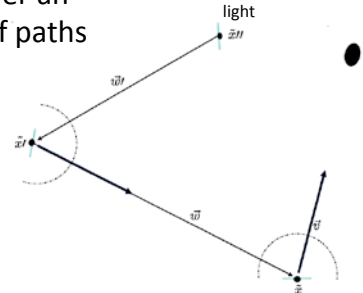
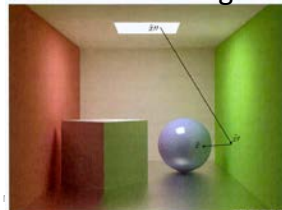


Min H. Kim (KAIST)

6

## Path tracing

- Adding one more bounce  $L^e(\tilde{x}, \tilde{v}) + L^1(\tilde{x}, \tilde{v}) + L^2(\tilde{x}, \tilde{v})$
- Geometric path of light transport:  $(\tilde{x}, \tilde{x}', \tilde{x}'')$
- Not as a nested integral over two hemispheres, but as an integral over an appropriate space of paths  $\rightarrow$  Path tracing



1

7

## Rendering equation


- Reflection equation in shorthand  $L^1 = BL^e$  where  $B$  is the bounce operator of light.
- Recursive light transport is  $L^{i+1} = BL^i$
- We can say  $L^1 = L^e + L^1 + L^2 + L^3 + \dots = L^e + B(L^1 + L^2 + L^3 + \dots) = L^e + BL^1$
- The rendering equation (integral equation) is

$$L^1(\tilde{x}, \tilde{v}) = L^e(\tilde{x}, \tilde{v}) + \int_H f_{\tilde{x}, \tilde{n}}(\tilde{w}, \tilde{v}) L^1(\tilde{w}, \tilde{x}) \cos(\theta) d\omega$$

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012


8



Chapter 23


## ANIMATION

Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012* 9




## Skinning

- Start with complicated mesh we want to animate
- Rigging: design a geometric hierarchical skeletal structure
- Associate each vertex with one “bone” (RBT node)
- Idea: express the vertex in its bone frame
- Manipulate the skeleton
- Vertices now move along with the bones




Min H. Kim (KAIST) *Foundations of 3D Computer Graphics, S. Gortler, MIT Press,*




## Details

- Instead of actually changing the vertex coordinates,
- A rest accumulated matrix represents the relationship between the object and rest bone frame.
- A new accumulated matrix represents the relation after the bone has been moved.
- Use these matrices, in the vertex shader to move the vertex.




Min H. Kim (KAIST) 11



## Details

- In this context, we can associate a vector of bone-weights and do soft skinning.
  - We use the weights to blend the updates.
- This is used extensively in games and would make a nice project



Min H. Kim (KAIST) 12

## Skinning (rigging) example



Aleka McAdams, Yongning Zhu, Andrew Selle, Mark Empey, Rasmus Tamstorf, Joseph Teran, and Eftychios Sifakis. 2011. Efficient elasticity for character skinning with contact and collisions. In *ACM SIGGRAPH 2011 papers* (SIGGRAPH '11), Hugues Hoppe (Ed.). ACM, New York, NY, USA, Article 37, 12 pages.

13

## Simulation

- Physics uses equations to describe physical processes.
- We can try to simulate these processes computationally.
- Techniques: physics and computational mathematics.
- Some methods are slow and only work for offline animation.
- Some methods can be made real-time
- Hard to control the output

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

14

## Particle simulation systems

- Simplest version of physics.
- A large bunch of non-interacting particles.
- Ordinary differential equation (ODE) for the time evaluation of a point:  

$$f = ma = m\dot{v} = m\ddot{x} \quad p = mv$$
- Force might be gravity or wind
- Can model flowing fall of water particles, or a stream of smoke particles in the air
- Typically each particle is rendered as a semitransparent little blob or surface

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

15

## ODE integration

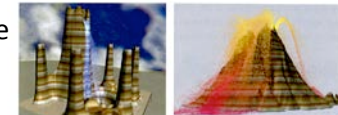
- Starting from an initial condition, we can discretize this ODE and march forward in time using so-called *Euler steps*:

$$x_{t+h} = x_t + v_t h$$

$$v_{t+h} = v_t + a_t h$$

$$a_{t+h} = f(x_{t+h}, t+h) / m$$

- Steps must be small (often need many more than 30/sec).
- There is a whole literature of more sophisticated ways to solve an ODE.



Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

16

## Particle simulation example



Tobias Pfaff, Nils Thuerey, Jonathan Cohen, Sarah Tariq, and Markus Gross.  
2010. Scalable fluid simulation using anisotropic turbulence particles. In ACM  
SIGGRAPH Asia 2010 papers (SIGGRAPH ASIA '10). ACM, New York, NY, USA, ,

Min H. Kim (KAIST) Article 174, 8 pages. Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

17

## Rigid Bodies

- Upgrade from particles to solid hard finite objects (dice rolling on a table)
- Need to deal with rotational issues
- With to deal with interaction: collision detection
  - Bounding hierarchies
  - Must undo interpenetration
  - Must have the object bounce (this requires hacked physics since real objects slightly deform and undeform)
- Must deal with objects resting on objects and not endlessly bouncing

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

18

## Cloth

- Can be modeled as a grid of particles connected by springs
- Can be modeled as mesh of physical triangular elements
- Need forces to avoid stretching and shearing and oscillation

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

19

## Cloth



Zhili Chen, Renguo Feng, and Huamin Wang. 2013. Modeling friction and air effects  
between cloth and deformable bodies. ACM Trans. Graph. 32, 4, Article 88 (July 2013), 8  
pages.

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

20

## Hair



- Hair modeling is also often similarly dealt with as a mass-spring model



Aleka McAdams, Andrew Selle, Kelly Ward, Eftychios Sifakis, and Joseph Teran. 2009. Detail preserving continuum simulation of straight hair. In ACM SIGGRAPH 2009 papers (SIGGRAPH '09), Hugues Hoppe (Ed.). ACM, New York, NY, USA, Article 62, 6 pages.

21

## Deformable materials



- Real objects are deformable
- Can be modeled as volumetric objects (mesh of 3D tetrahedra)



Chris Wojtan, Nils Thürey, Markus Gross, and Greg Turk. 2009. Deforming meshes that split and merge. In ACM SIGGRAPH 2009 papers (SIGGRAPH '09), Hugues Hoppe (Ed.). ACM, New York, NY, USA, Article 76, 10 pages.

22

## Fire and water



- Special physical equations
- Modeled with combination of equations



Jeffrey N. Chadwick and Doug L. James. 2011. Animating fire with sound. In ACM SIGGRAPH 2011 papers (SIGGRAPH '11), Hugues Hoppe (Ed.). ACM, New York, NY, USA, , Article 84, 8 pages.

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

23

## Human locomotion



- Not passive objects
- Much harder than previously discussed phenomenon
- Ideas are used from robotics, control, and optimization
- Nowadays motion capture data is relied on heavily, and possibly altered or used as part of the rocket science.

Min H. Kim (KAIST)

Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

24

## Human locomotion



Jack M. Wang, David J. Fleet, and Aaron Hertzmann. 2010. Optimizing walking controllers for uncertain inputs and environments. In ACM SIGGRAPH 2010 papers (SIGGRAPH '10), Hugues Hoppe (Ed.). ACM, New York, NY, USA, Article 73, 8 pages.<sup>25</sup>