CS380: Introduction to Computer Graphics
Color (2/4)
Chapter 19

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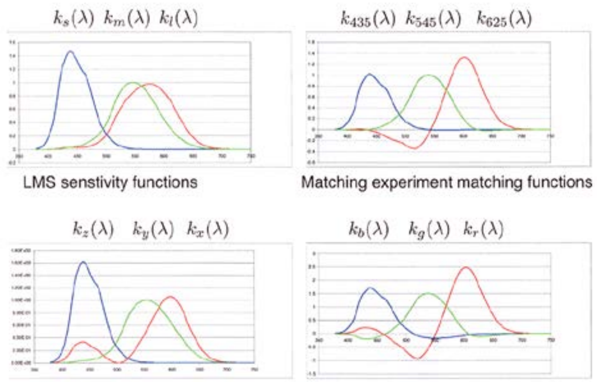
Color

SUMMARY

Cones



LMS senstivity functions | Matching experiment matching functions

XYZ matching functions | RGB matching functions

Mixed beams

- For a mixed beam of light $l(\lambda)$, the three responses $[L, M, S]^t$ are:

$$L = \int_\Omega l(\lambda) k_l(\lambda) d\lambda$$

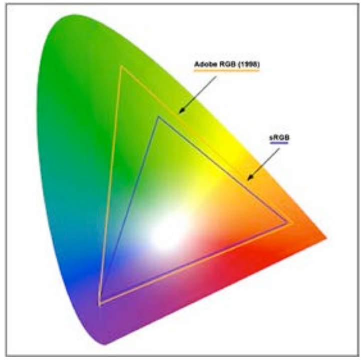
$$M = \int_\Omega l(\lambda) k_m(\lambda) d\lambda$$

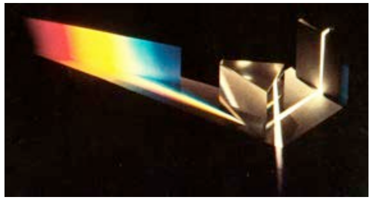
$$S = \int_\Omega l(\lambda) k_s(\lambda) d\lambda$$

where $\Omega=[380...770]$

Example

- Qualitative comparison of color gamuts of different devices or color spaces
- sRGB vs. Adobe RGB





Chapter 19

COLOR (2/4)

Bases

- We can insert any (non singular) 3-by-3 matrix *M* and its inverse to obtain:

$$\vec{c}(l(\lambda)) = \left(\begin{bmatrix} \vec{c}(l_{436}) & \vec{c}(l_{546}) & \vec{c}(l_{700}) \end{bmatrix} M^{-1}\right)\left(M \begin{bmatrix} \int_\Omega k_{436}(\lambda) l(\lambda) d\lambda \\ \int_\Omega k_{546}(\lambda) l(\lambda) d\lambda \\ \int_\Omega k_{700}(\lambda) l(\lambda) d\lambda \end{bmatrix}\right)$$

$$= \begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{bmatrix} \begin{bmatrix} \int_\Omega k_1(\lambda) l(\lambda) d\lambda \\ \int_\Omega k_2(\lambda) l(\lambda) d\lambda \\ \int_\Omega k_3(\lambda) l(\lambda) d\lambda \end{bmatrix}$$

Bases

- Where the $\vec{c}_i$ describe a new color basis defined as $\begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{bmatrix} = \begin{bmatrix} \vec{c}(l_{436}) & \vec{c}(l_{546}) & \vec{c}(l_{700}) \end{bmatrix} M^{-1}$
- The *k(λ)* functions form the new associated matching functions, defined by:

$$\begin{bmatrix} k_1(\lambda) \\ k_2(\lambda) \\ k_3(\lambda) \end{bmatrix} = M \begin{bmatrix} k_{436}(\lambda) \\ k_{546}(\lambda) \\ k_{700}(\lambda) \end{bmatrix}$$

Basis specification

- Starting from any fixed basis for color space, such as $\left[\vec{c}(l_{436}) \quad \vec{c}(l_{546}) \quad \vec{c}(l_{700}) \right]$
- 1: specify an invertible 3-by-3 matrix *M.*
- 2: specify three actual colors $\vec{c}_i$
  - Each such $\vec{c}_i$ can be specified by some light beam $l_i(\lambda)$ that generates it.
  - Plug each such light beam into above calculation to obtain its 456 color coordinates, determining the matrix.

Basis specification

- Directly specify three new matching functions
  - To be valid matching functions, they must arise from a basis change like the above equation, and so each matching function must be some linear combination of $k_{436}(\lambda)$, $k_{546}(\lambda)$ and $k_{625}(\lambda)$
  - else we will not respect metamerism
  - Some cameras can mess this up

LMS revisited

- The LMS matching functions we saw originally describe a basis for color space.
- The coordinates of a color are called $[L, M, S]^t$
- The actual basis is made up of three colors we can call $[\vec{c}_l, \vec{c}_m, \vec{c}_s]$
- The color $\vec{c}_m$ is a very imaginary color.
- There is no real light beam with LMS color coordinates $[0,1,0]^t$

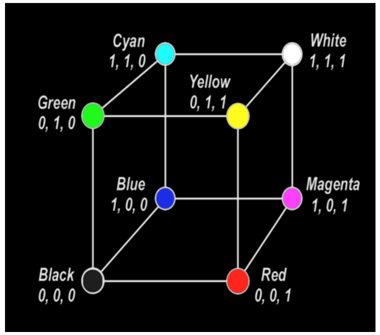
Gamut

- Observe: we cannot find three vectors that both hit the lasso curve and contain the entire curve in their positive span.
- So if we want a basis where all actual colors have non-negative coordinates, at least one of the basis vectors defining this octant must lie outside of the cone of actual colors.
  - Such a basis vector must be an imaginary color.
- Conversely, if all of our basis vectors are actual colors, and thus within the color cone, then there must be some actual colors that cannot be written with non-negative coordinates.
- In this basis, we say that such colors lie outside the gamut of this color space.

Remember This Color

Device Dependent Color Spaces

- Pros:
  - Simple description of color for the device
  - Natural, easy way to specify color to the user
- Cons:
  - Cannot compare colors between devices
  - Not perceptually uniform



HSV Color Space Math

- Value: $V = M = \max(R, G, B).$
- Saturation:

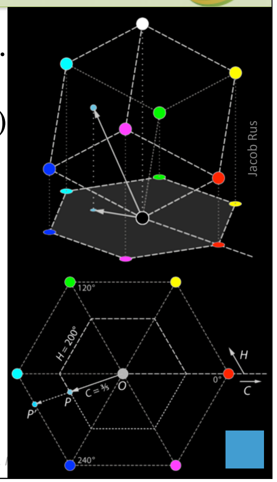
$$m = \min(R, G, B)$$

- Hue:

$$C = M - m,$$

$$S = C / V,$$

$$H = \begin{cases} 360 + 60(G - B)/C & \text{if } M = R \\ 120 + 60(B - R)/C & \text{if } M = G \\ 240 + 60(R - G)/C & \text{if } M = B \end{cases}$$

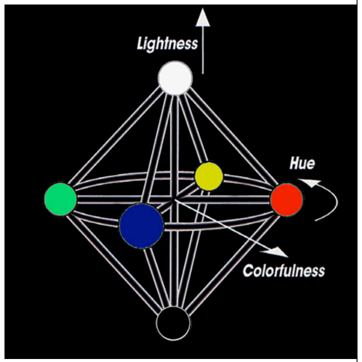


Device Independent Color Spaces

- Pros:
  - Based on human visual perception
  - Color specification independent of device
  - Interchangeable color among devices
  - Comparison, computation of small color differences
- Cons:
  - CIEXYZ: not uniform
  - CIELAB, CIELUV, CIEXYZ, Munsell: all dependent on the illuminant

Perceptual Color Models

- Opponent primaries
- Three dimensions: lightness, colorfulness, and hue (*L, C, H*)
- Related to processes of human visual perception
- Meaningful way of describing color



Munsell System (1915)

- Five primary hues:

Red	Yellow	Green	Blue	Purple

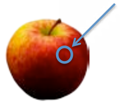
- Value range:

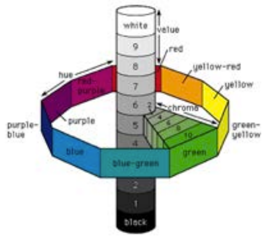
0	...	5	...	10

- Chroma range:

0	...	5	...	∞

10RP 4/10 = a specific reddish purple hue of 10RP, a value of 4, and a chroma of 10

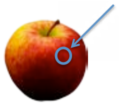


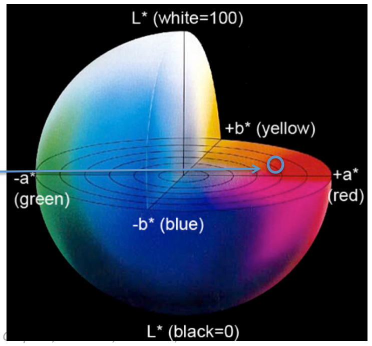


CIE Uniform Color Spaces (1976)

- Originated from Hunter Lab 1948
- Perceptually uniform color definition
- Driven from CIEXYZ

L*= 43.31
a*= 47.63
b*= 14.12





CIE LAB Math

- Simplified cone response (XYZ and a cubic root func.)
- Color opponent theory for computing chroma and hue
- Lightness:
- Color opponents:

$$L^* = 116(Y/Y_n)^{1/3} - 16,$$
$$a^* = 500[f(X/X_n) - f(Y/Y_n)],$$
$$b^* = 200[f(Y/Y_n) - f(Z/Z_n)],$$

$$f(x) = \begin{cases} x^{1/3}, & x > 0.008856 \\ 7.787x + 16/116, & x \le 0.008856 \end{cases}$$

- Chroma:

$$C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2},$$

- Hue:

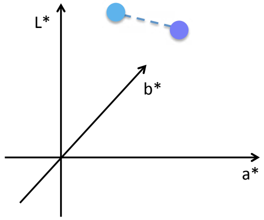
$$h_{ab} = \tan^{-1}(b^*/a^*).$$

Remember This Color

 Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012

Color Differences CIE ΔE^*_{ab}

- Conventional Euclidean metric in a perceptually uniform color space (CIELAB)



$$\Delta E^*_{ab} = \sqrt{\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2}$$

 Foundations of 3D Computer Graphics, S. Gortler, MIT Press, 2012