

CS 380

Introduction to Computer Graphics

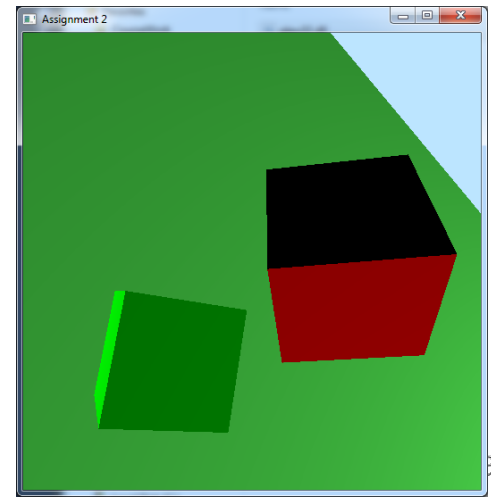
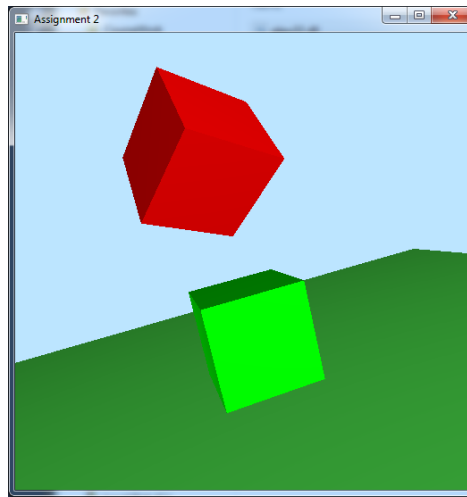
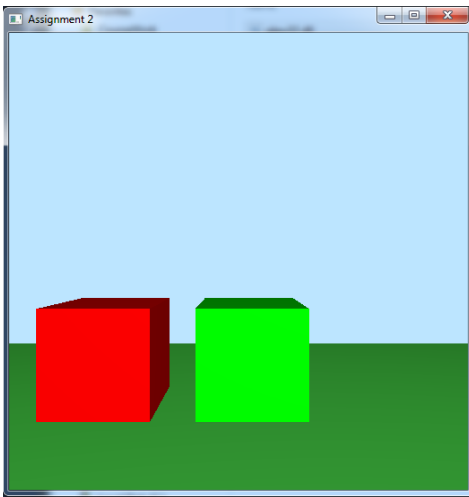
LAB (3)

2018.03.26

- Note that we will not teach the answer of this homework, as the homework must be done by yourself.
- Instead, we will explain the necessary background which is required to finish this HW.
- Some part of this slides are from lecture notes of this course.
- If you have problem with `std::min`, add `#include <algorithm>` in `cvec.h`

Goals

1. Draw two cubes
 2. Create different viewpoints
 3. Move the objects freely w.r.t. the current viewpoint frame
- We have **four tasks** to complete in this hw2.



Affine Transformation

Full affine transformation

Translation

Rotation

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & h \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & 0 \\ e & f & g & 0 \\ i & j & k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} l & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} i & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = TL$$

- We are transforming a point \tilde{p} in a frame $\vec{\mathbf{f}}^t$

$$\tilde{p} = \vec{\mathbf{f}}^t \mathbf{c}$$

- With a matrix

$$\mathbf{S} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the stretches by factor of two in first axis of $\vec{\mathbf{f}}^t$

- Performing a transform: $\vec{\mathbf{f}}^t \mathbf{c} \Rightarrow \vec{\mathbf{f}}^t \mathbf{S} \mathbf{c}$

- Suppose another frame: $\vec{\mathbf{a}}^t = \vec{\mathbf{f}}^t \mathbf{A}$

- We could express the point with a new coordinate vector

$$\tilde{p} = \vec{f}^t \mathbf{c} = \vec{a}^t \mathbf{d}$$

$$\vec{f}^t \mathbf{c} = \vec{f}^t A \mathbf{d}$$

$$\mathbf{d} = A^{-1} \mathbf{c}$$

$$\vec{a}^t = \vec{f}^t A$$

$$\vec{f}^t = \vec{a}^t A^{-1}$$

- Now S transforms the point \tilde{p} with respect to \vec{a}^t

$$\vec{a}^t \mathbf{d} \Rightarrow \vec{a}^t S \mathbf{d}$$

- Point is transformed **with respect to** the the frame that appears immediately to the left of the transformation matrix in the expression.

- We read

$$\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t S$$

$\vec{\mathbf{f}}^t$ is transformed by S with respect to $\vec{\mathbf{f}}^t$

- We read

$$\vec{\mathbf{f}}^t = \vec{\mathbf{a}}^t A^{-1} \Rightarrow \vec{\mathbf{a}}^t S A^{-1}$$

$\vec{\mathbf{f}}^t$ is transformed by S with respect to $\vec{\mathbf{a}}^t$

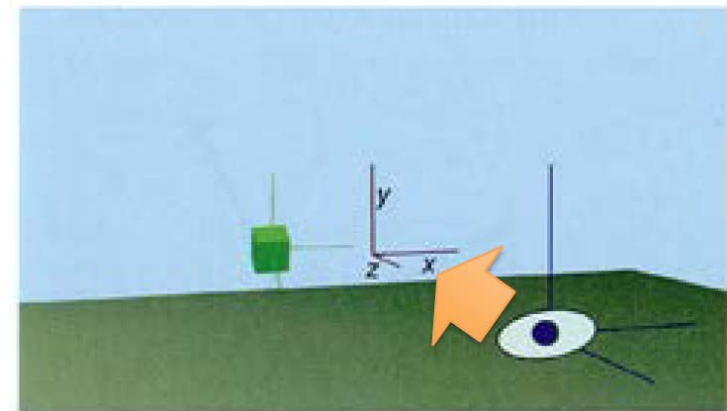
- World frame \vec{w}^t
 - an absolute frame in 3D space
 - All other frames are represented by this frame

- Object frame \vec{o}^t
 - All objects should have own frame

$$\vec{o}^t = \vec{w}^t O$$

- Eye frame \vec{e}^t

$$\vec{e}^t = \vec{w}^t E$$



(a) The frames

Eye Coordinate

- we explicitly store the matrix E

$$\vec{e}' = \vec{w}' E$$

$$\tilde{p} = \vec{o}' \mathbf{c} = \vec{w}' O \mathbf{c} = \vec{e}' E^{-1} O \mathbf{c}$$

– Object coordinates: \mathbf{c}

– World coordinates: $O \mathbf{c}$

– Eye coordinates: $E^{-1} O \mathbf{c}$

MVM = $\text{inv}(\text{eyeRbt}) * \text{objRbt};$
(Model View Matrix)

– Calculating the eye coordinates of every vertexes:

$$\begin{bmatrix} x_e \\ y_e \\ z_e \\ 1 \end{bmatrix} = E^{-1} O \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

Task1. Draw Two Cubes

- A frame needs to be defined to the each object.
- A frame of a red cube

$$\vec{O}_R^t = \vec{W}^t O_R$$

World frame

Transformed frame

Affine transformation

- A frame of a green cube

$$\vec{O}_L^t = \vec{W}^t O_L$$

- linFact
 - Rotation matrix: 4 x 4
- transFact
 - Translation matrix: 4 x 4
- $M = \text{transFact}(M) * \text{linFact}(M)$
- By using these two function, you can obtain the full **affine transformation matrix**

Task3. Change Viewpoint

- Now, we have a pre-defined view point, which is called eye-view.
- The rendered image is generated in the frame of the eye-view.
- Our job is to change the eye-view according to the input of the user **by modifying the eye-view matrix**. (sky, cube1, cube2)

Task 4: Manipulation Mode

- If 'o' key is pressed, the object that we can manipulate should be changed (sky, cube1, cube2)
- You should choose different frames carefully depending on object and eye mode (8 combinations)
- In order to complete task4, you should utilize a transformation w.r.t. a frame

Task 4: Manipulation Mode

\vec{a}^t should be the cube-sky frame

Manipulated object

Eye frame

Sky camera

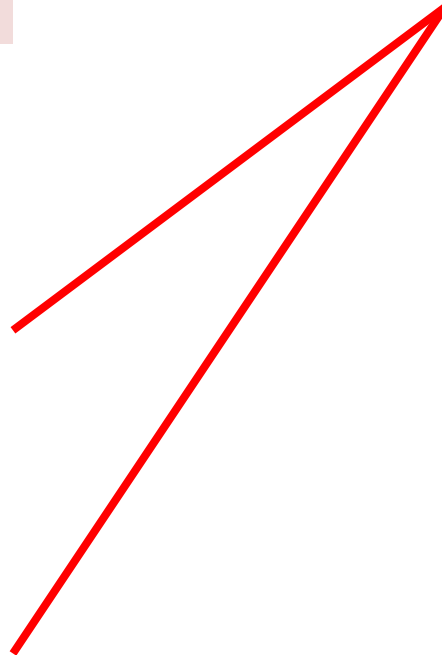
Sky camera

Cube 1

Cube 1

Cube 2

Cube 2



Task 4: Manipulation Mode

\vec{a}^t should be the cube-cube frame

Manipulated object

Eye frame

Sky camera

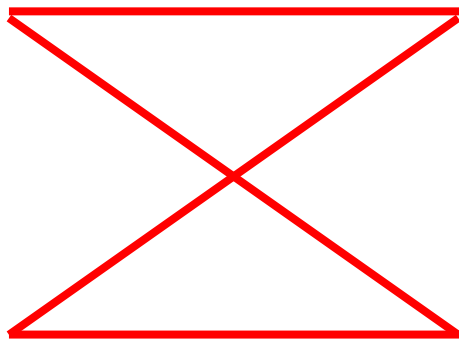
Sky camera

Cube 1

Cube 1

Cube 2

Cube 2



Task 4: Manipulation Mode

\vec{a}^t should be the world-sky frame or sky-sky frame

change mode by 'm'

Manipulated object

Eye frame

Sky camera



Sky camera

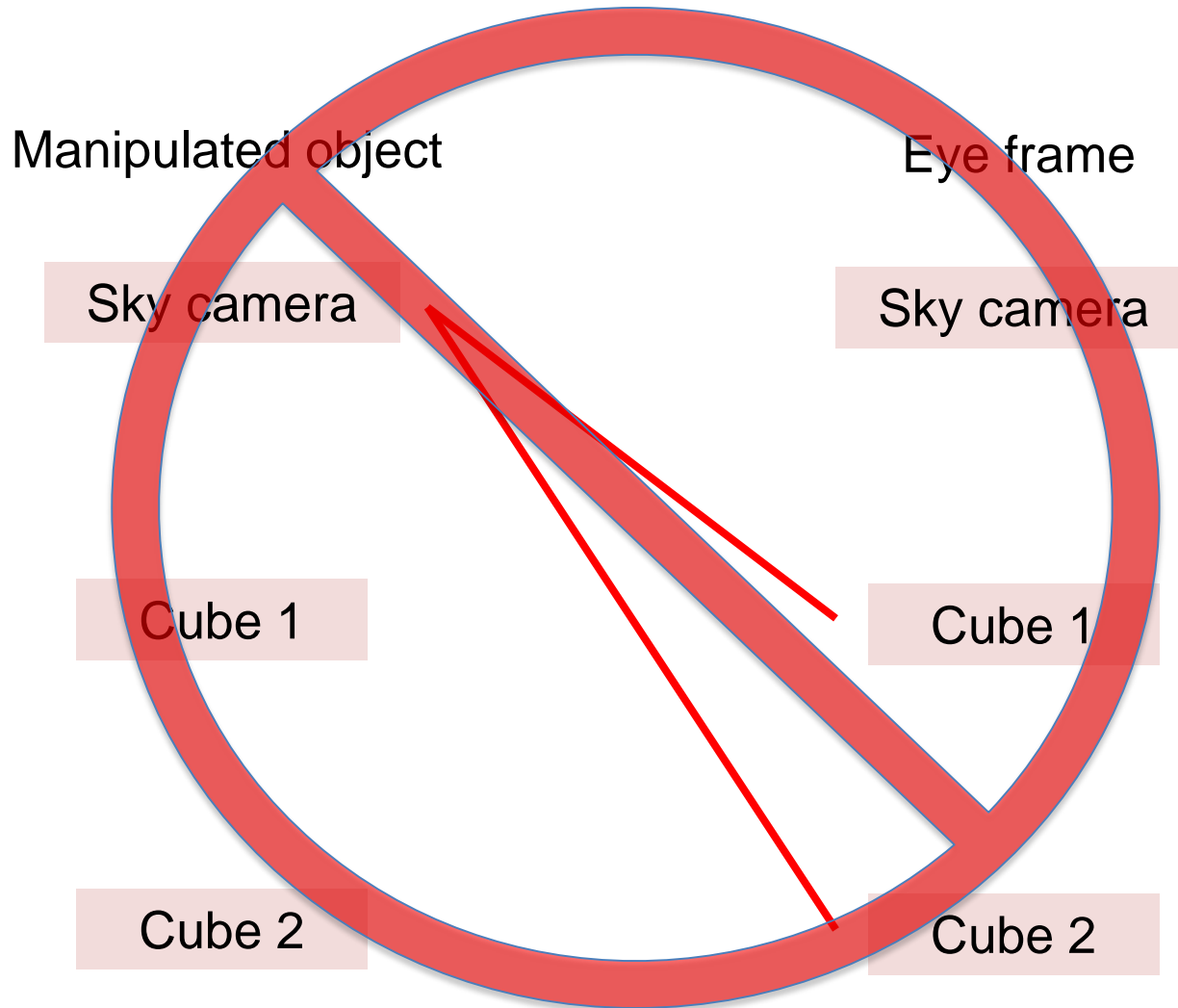
Cube 1

Cube 1

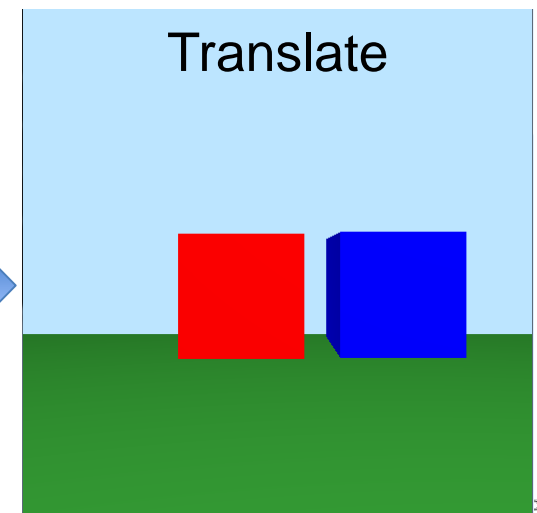
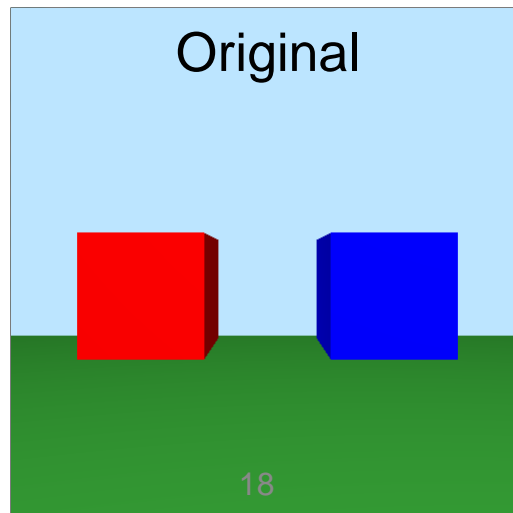
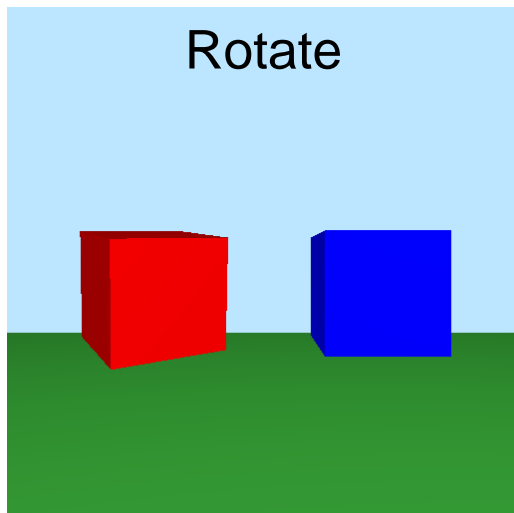
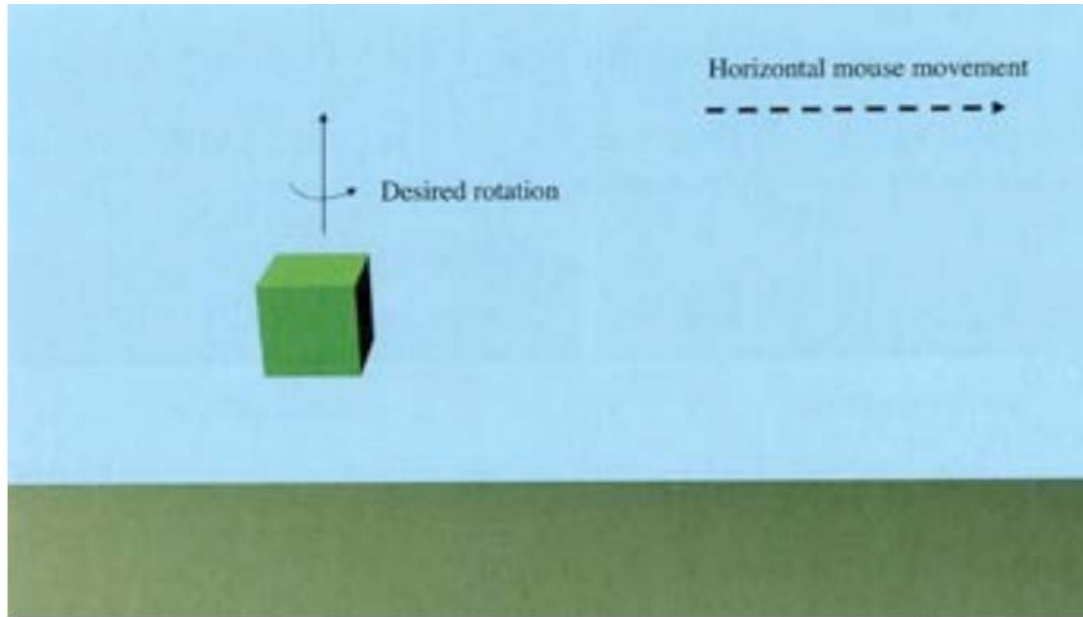
Cube 2

Cube 2

Task 4: Manipulation Mode



Task 4: Move Objects



Task 4: How to choose the frame?

- Translation : direction
- **Rotation : position (origin) + direction (axis)**

- Recalling the Affine transform.: $A = TR$
- The object's Affine transform.: $O = (O)_T (O)_R$
- The eye's Affine transform.: $E = (E)_T (E)_R$

$$\vec{\mathbf{a}}^t = \vec{\mathbf{w}}^t (O)_T (E)_R$$